DwightGrantConsulting 7332 Eads Ave. La Jolla CA 92037 415-509-3943

Dwight Grant, PhD
dwight@dwightgrantconsulting.com

Valuing Interest Rate Derivatives and Hedging in a Low Interest Rate Environment

by

Marina Kagan and Dwight Grant

Valuing Interest Rate Derivatives and Hedging In a Low Interest Rate Environment[1]

In the era of low interest rates, firms may enter into a variable rate loan with a base rate such as one-month LIBOR plus a spread but with a floor of 0% on the base rate. If the at the same time the firm hedges the LIBOR risk by entering into pay-fixed-receive-LIBOR swaps with no floor on LIBOR. This raises two issues.

1) What is the value of the LIBOR floor and

2) If the swaps do not have a floor, how does the floor affect whether the swaps qualify for hedge accounting?

Welcome to the brave new world of negative interest rates. Until recently, negative interest rates – the lender pays the borrower – were considered, if they were considered at all, a theoretical curiosity. Not so today. Central banks, in an effort to stimulate economic activity and lending by commercial banks, are imposing negative interest rates on excess reserves. These negative rates have fed into the inter-bank lending markets and the swap markets. Japanese, Swiss and Euro swap curves are all in negative territory for terms of up to 5 years. U.S. rates are low but currently positive.

Interest rate floors do have value the absence of a floor in the swap affects the calculations necessary to support hedge qualification. The floor in the bank loan creates embedded interest rate puts with a strike or exercise price of 0%. These embedded derivatives must be valued and incorporated in the hedge effectiveness testing. The swaps were still likely to be effective hedges but the floor would introduce additional ineffectiveness. Valuing puts with a strike price of 0% is a new and challenging area of practice.

The traditional approach to valuing interest rate derivative contracts such as caps, floors, and swaptions, has been to use the Black model.[2] The Black model is a variation on the famous Black-Scholes model and it assumes that interest rates cannot be zero or negative. In technical terms, it assumes that the distribution of interest rates follows a lognormal distribution. The lognormal distribution is similar to the relatively familiar normal distribution or what is often called the bell-shaped curve, except that the normal distribution does allow negative values. We will have more to say about the normal distribution a little later in this note.

Some people have tried to finesse the problem by using a “shifted-lognormal model”. Under this approach, both the interest rate probability distribution and the strike price (floor) of the option are shifted up. For example, if the interest rate is 1.0% and the floor is 0.0%, they might both be increased by 0.5%. You then use the Black formula to value the floor *as if* the interest rate were 1.5% and the floor were 0.5%. A

[1] Marina is a Director in the Deal Practice at PwC. We completed this work when we were colleagues at PwC.
[2] F. Black. The Pricing of Commodity Contracts. Journal of Financial Economics, 3, 1976.

significant drawback of this model is that the shift parameter is subjective and significantly influences the price of the put. We illustrate a second concern with an example of its performance. We used the Black model to value a five-year interest rate floor with an interest rate of 3%, a floor of 2% and a volatility of 20%. We also valued the floor *as if* the interest rate were 3.5% and the floor were 2.5%, a 0.50% shift. The shifted-lognormal model produces a price that is 70% higher than the regular Black model[3]. That result suggest the lognormal-shift model may not be reliable.

Fortunately, there is a better solution found in a PhD thesis written by Louis Bachelier at the Sorbonne in 1900.[4] Bachelier was the first to apply advanced mathematics to finance and, in particular, the valuation of options. He developed a valuation formula very similar to that discovered by Black and Scholes seven decades later. The important difference in their discoveries was that Bachelier assumed a normal distribution, which is just what we need when valuing interest rate options in the current low interest rate environment.

Two issues arose as we applied Bachelier's formula. First, in its usual implementation it does not adjust for what is known as the convexity feature of fixed income payments. This tends to overestimate the value of an interest rate floor. Second, it does not discount future cash flows by the interest rates that produce the cash flows. This tends to underestimate the value of an interest rate floor. We investigated these two issues using a more complex model and found that these two effects tend to offset each other. Thus, Bachelier's formula is an effective way to value interest rate derivatives in this low interest rate environment. We find it a very efficient and practical way to value floors in bank loans and to make the calculations necessary to support hedge documentation.

We mentioned using a more complex model to test the effects of the two minor shortcomings in the Bachelier model. It is worth noting that there are many alternatives when it comes to modelling interest rates. One well known model authored by Hull and White also assumes normally distributed interest rates, but allows rates to move back toward a "normal" level. This is an attractive feature if you believe that there is limit on how low interest rates can go. There are also more complex versions of the shifted-lognormal model. Ultimately, the choice of a model depends on the objective of the project. We have experience implementing a wide variety of interest rate models and always aim for a best practical and supportable solution. For simple 0% interest rate floors, the Bachelier model is likely simpler to implement and very likely to be sufficient.

---

[3] We note that the value differential described above is calculated by applying the same volatility in the shifted-lognormal calculation as in the original calculation. It is, indeed, possible, to solve for a shifted-lognormal volatility, which would make the option values identical. To do so, however, you must first know the true value of the floor. If you knew that, you would not be valuing it in the first place. In addition this "new volatility" would no longer accurately represent the volatility of the interest rate process, and so its use for pricing more exotic options is problematic.

[4] L. Bachelier. *Théory de la Spéculation*. Annales Scientiques de l'École Normale Superieure, 1900.