DWIGHTGRANTCONSULTING 7332 EADS AVE. LA JOLLA CA 92037 415-509-3943

Dwight Grant, PhD
dwight@dwightgrantconsulting.com

Applying the Option Pricing Method to Discrete Payoffs in Complex Capital Structures

by

Dwight Grant

Applying the Option Pricing Method to Discrete Payoffs in Complex Capital Structures[1]

Contingent Claims Analysis or Option Pricing Method (OPM) is a popular way of allocating equity value among securities in a capital structure. Its popularity is based on its capacity to model very complex securities, the need to estimate only two judgmental inputs, equity value and volatility and its use of valuation formulas that are easy to implement, namely the Black-Scholes-Merton (BSM) option pricing formulas. The OPM necessarily relies on the assumptions underlying BSM formulas. Investors and appraisers have criticized the OPM because, as it is typically presented, one of the BSM assumptions is that equity value is lognormally distributed. In particular, they argue that the lognormal distribution is inappropriate for early stage companies for which outcomes are more accurately represented by a small number of discrete outcomes, including in the extreme, two outcomes, success or failure.[2]

This criticism is misguided but it does highlight that option pricing methodology is not being used to its full potential in allocating equity value. There are almost certainly cases where the future values of equity can most accurately be represented by discrete values, including as few as two values. We demonstrate how option pricing methodology can be used to value securities in a complex capital structure with only two future equity values. We then consider distributions with four outcomes to illustrate the method's flexibility. In both cases, we also show that the values produced by the discrete distributions can be very closely matched by the OPM. Consequently the OPM may be a reasonable way to allocate value, even if the true distribution of future values is discrete.

Option Pricing with Two Liquidity Event-Date Values

Option pricing employs two concepts: no-arbitrage pricing and replicating portfolios. No-arbitrage pricing means that two things that are identical cannot sell at different prices. That is an obvious idea. If the prices are different, traders will buy at the low price and sell at the high price to make a profit. Traders will continue this activity until the two prices converge.[3] A replicating portfolio refers to the idea that you can create a portfolio of securities that has the same payoffs as a single security. If you can do that, then the no-arbitrage principle requires that the price of the portfolio and the price of the security it replicates must be the same.

These two general concepts allow us to value securities in a complex capital structure relative to the value of the related equity and a risk-free bond. To do that, we assume that you can buy or sell both the related equity and the risk-free bond. In the case of the related equity, that means take a long or short position in it. In

[1] Please do not quote or reference without permission.

[2] See for example pages 33 – 4 in Steven Nebb and David Larsen. "10 Years after PEIGG, is the world a better place? Venture Capital Review Issue 29, 2013, pp. 29 – 35.

[3] If it costs to transact then there can be a bid-ask spread for buying and selling prices. Similarly, if the two alternatives are located in different places, there can be price differences based on transportation costs.

the case of the risk-free bond, it means lend or borrow at the risk-free rate. These are standard option pricing assumptions.

At this point in the discussion, the risky asset is a share of a common stock. We assume the price of the stock is 1.00 and one period from now it will be worth either 1.25 in the "up-state" or 0.80 in the "down-state". The risk-free bond costs 1.00 and it will be worth 1.05 in one period in both the up- and down-states.

Suppose that there is a third security whose price we do not know. This security will be worth 0.625 in the up-state and 0.400 in the down-state. What is it worth today? Because its payoffs in one year are exactly one-half those of the stock, it must be worth 0.50 today: Two shares of this security are identical to one share of the common stock so this security must be worth one-half the value of the common stock. Similarly, if there is a security that offers 0.525 in the up-state and 0.525 in the down-state, it must be worth 0.50 because its payoffs are identical to one-half the payoffs of the risk-free security. We now consider 6 examples that involve both the common stock and the bond. See Table 1 for the payoff diagrams and the values of each of the example securities.

Example 1

Suppose a security offers 2.30 (1.25 + 1.05) in the up-state and 1.85 (0.80 + 1.05) in the down-state. What is it worth? Again, by inspection we can see that it offers the same payoffs as one share of the stock and one bond. Therefore it must be worth 2.00.

Example 2

Here is a question that we cannot answer by inspection: What is the no-arbitrage price of a security that pays 1.4625 in the up-state and 1.1250 in the down-state? To answer this, we need to build a replicating portfolio and this requires basic algebra. Let the number of units of the stock in a portfolio that replicates these payoffs be x and the number of the units of the bond in the portfolio be y. Now,

$1.25x + 1.05y = 1.4625$ and

$0.80x + 1.05y = 1.1250$

We have two equations in two unknowns and we can solve for x and y.

Subtract the second equation from the first:

$0.45x = 0.3375$ and $x = 0.75$.

If we substitute that value into the first equation:

$1.25(0.75) + 1.05y = 1.4625$ and $y = (1.4625 – 1.25(0.75))/1.05 = 0.50$;

the security must cost

$$1.00x + 1.00y = 1.00(0.75) + 1.00(0.50) = 1.25,$$

because its payoffs are identical to 0.75 shares of stock and 0.50 bonds.

Example 3

Suppose a security offers 1.975 in the up-state and 1.075 in the down-state. Repeating the previous analysis, we find $x = 2.0$ and $y = -0.50$. We interpret this to mean that you must buy 2.0 shares of stock and finance their purchase in part by selling short 0.5 units of the bond, which is the same as borrowing 0.50 at the risk-free rate. This security is worth:

$$1.00x + 1.00y = 100(2.0) + 100(-0.5) = 1.50$$

Example 4

Now, we value a call option on the common stock with an exercise price of 1.10. This call option pays 0.15 in the upstate and 0.00 in the down-state. If we repeat the algebra we find $x = 0.333$, $y = -0.254$ and the call option is worth 0.0794.

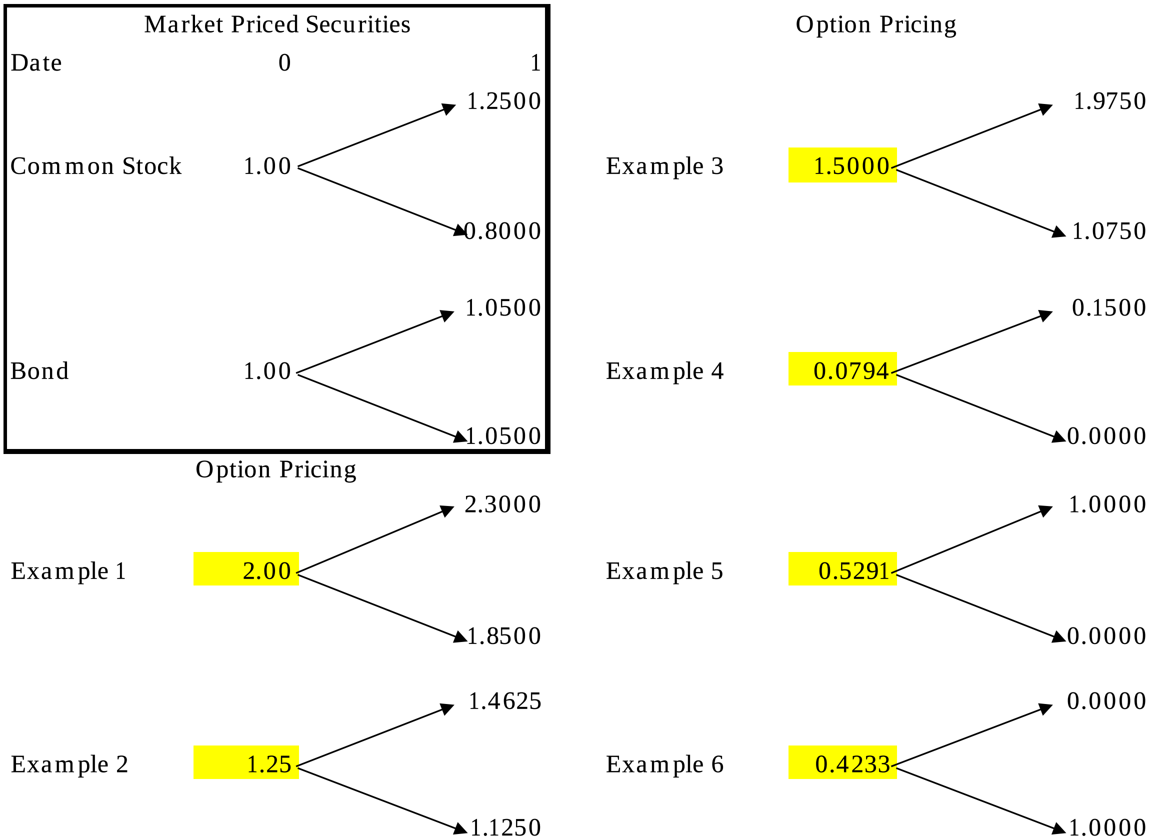
Examples 5 and 6

Lastly, a security that pays 1 in the up-state and 0 in the down-state is priced at 0.5291 and a security that pays 0 in the up-state and 1 in the down-state is priced at 0.4233. These prices are especially useful because we can value any set of payoffs simply by multiplying the payoffs by those values. Referring to example 3,

$$0.5291(1.975) + 0.4233(1.075) = 1.5000.$$

Note that we can price the various securities, without knowing the actual probability of future payoffs for the common stock or its expected rate of return. This is possible because the probabilities and expected return are embedded in the current common stock price.

Table 1
Examples of Relative/Option Pricing



We now apply this logic to valuing the securities in a complex capital structure. The details of the example are in Table 2.

Table 2
Capitalization Table

	Preferred	Common	Options
Face	130		
Coupon	7.00%		
Strike price	3.09825		1
Number of common shares when exercised	55.00	50	10
Initial Equity	200		
Risk-free rate	1.00%		
Term	4		
Equity exit value (high)	600		
Equity exit value (low)	80		

Again, as is customary, we assume no interest or coupons are paid until the exit and accumulated interest is lost if conversion occurs. The preferred shares convert into 55 common shares giving them an effective conversion price of

$3.09825 = 130(1.07)^4/55$. Following the valuation process described above for binary outcomes, we reach the conclusions shown in Table 3.

We report the waterfall results for each security for each of the two payoffs, 600 and 80 and then value the payoffs for each security using the values of 1 unit in each of the up-and down-states, 0.237 and 0.724 respectively. We calculated these two values exactly as described in examples 5 and 6.

Table 3

Payoffs at the Liquidity Event and Values

	Total Payoff	State	Security Payoffs: Preferred	Security Payoffs: Common	Security Payoffs: Options	Value per Unit
Year 0	4					
	600.00	up	310.9	242.6	46.5	0.237
200.00						
	80.00	down	80.0	0.0	0.0	0.724
	Value at time 0		131.5	57.4	11.0	0.961
Total				200.000		

The values of the securities are their payoffs in each of the “u” and “down” states multiplied by the value of one unit in each case, 0.237 for the up state and 0.724 for the down state.

Preferred = 0.237(310.9) + 0.724(80) = 131.5
Common = 0.237(242.6) + 0.724(0) = 57.4
Preferred = 0.237(46.5) + 0.724(0) = 11.0

Before we move on to the next example, we ask the following question. What would the values of the securities be if we assume that the distribution of future payoffs is lognormal? The answer depends upon the volatility. For discussion purposes we report in Table 4 the values of the securities if the volatility is 62% and compare them with the values when the equity follows a binomial distribution.

Table 4

	Equity	Preferred	Common	Options
Binary distribution	200.00	131.54	57.44	11.02
Lognormal distribution (vol = 62%)	200.00	131.06	57.96	10.99

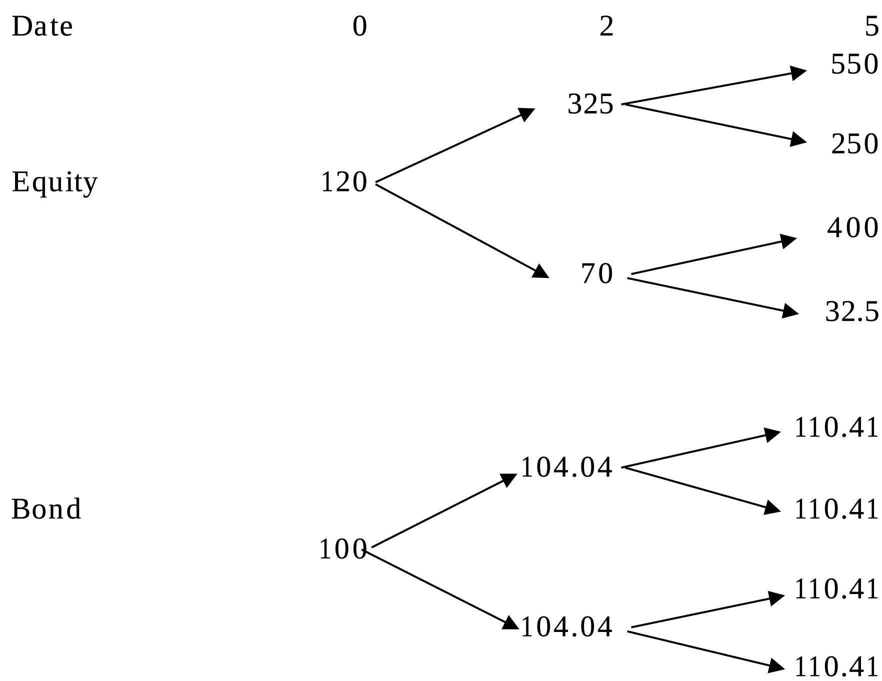
The work leading up to Table 3 demonstrates how you can use option pricing methodology when there are only two future payoffs. Therefore, the claim that the OPM requires an assumption of a lognormal distribution is in error. Table 4 goes one step further. It demonstrates that an OPM allocation of value assuming a lognormal distribution can provide an excellent approximation of the value produced assuming only two payoffs. The allocation of value in the case of the binary distribution depends on the subjective determination of the two possible outcomes; similarly, the allocation of value with the assumption of a lognormally distributed distribution depends on the subjective determination of the volatility. In this comparison, we selected two subjective distributions that produced virtually identical allocations of value. This supports a view that the lognormal distribution assumption does not necessarily provide an inaccurate allocation of the equity value of early stage companies.

Option Pricing with Four Liquidity Event Date Values

In this illustration, the initial equity value is 120 and the evolution of the equity value is described in the tree below. After 2 years the value of equity is forecast to have either increased to 325, the up-state value or decreased to 70, the down-state value. If it increased to 325 after 2 years, it is forecast to increase to either 500, the up-up-state value, or decrease to 250, the up-down-state value, after 3 more years. If equity initially decreased to 70 it is forecast to either increase to 400, the down-up-state value or to decrease to 32.5, the down-down-state value, after an additional 3 years. The risk-free rate of interest is 2% per year for 5 years.

Table 5

Two Securities Priced in the Market



To value securities with payoffs at year 5 we must repeat the earlier process of finding the no-arbitrage value of 1 unit when each of the payoffs, 550, 250, 400 and 32.50 occurs. While this may appear to be a daunting challenge it is only a little more difficult than the problem we have already solved. We use the same method and, as before, find the value at year 0 of 1 in the up-state and 1 in the down-state. We also must find the value at year 2 of 1 in each of 4 states: the up-up-state, the up-down-state, the down-up-state and the down-down-state. We will discuss two of the values in more detail and then report the valuation results in Tables 6 and 7.

The portfolio of securities at date o, that replicates a payoff of 1 in the up-state and 0 in the down state is a long position equal to 0.392% of the equity and a short position in the bond of 0.264%.[4] The value of this position at date 0 is 0.00392(120) – 0.00264(100) = 0.2067 and this is the value of 1 in the up-state. The portfolio of securities at date 2, that replicates a payoff of 1 in the up-up-state and 0 in the up-down-state is a long position of 0.333% in the equity and a short position of 0.755% in the bond. The value of this position is 0.00333(325) – 0.00755(104.04) = 0.2981. This is the value at year 2 of 1 in the up-up-state. The value at year 0 of 1 in the up-up-state is the product of these two values, (0.2067)(0.2981) = 0.0616.

Having shown how to use binomial option pricing to value 4 payoffs, we now apply these results to the valuation of securities in a capital structure composed of convertible preferred stock, common stock and options. The convertible preferred has a face value of 60 and an annual coupon of 2.5%. It is convertible into 30 common stock shares. There are 35.5 common stock shares outstanding and 14 options with an exercise price of 3.50. Table 8 displays to the payoffs at year 5 for each security and reports their values. The last row provides the values of the securities if we value them in an OPM with a volatility of 53.5%. As in the earlier example, the difference between the allocated values when there are a small number of discrete future values of equity, in this case 4 outcomes, and where the future values are lognormally distributed is trivial.

Summary

We demonstrate that, contrary to what has been claimed, 0ption pricing methods can be used to allocate equity value in a complex capital structure when there are a discrete number of future equity values. In addition, we show that this allocation can be replicated to a close approximation by the typical OPM which assumes future equity values are lognormally distributed. It is a matter of judgment which is more appropriate.

[4] This calculation is identical to that shown in example 5.

Table 6
Relative/Option Pricing

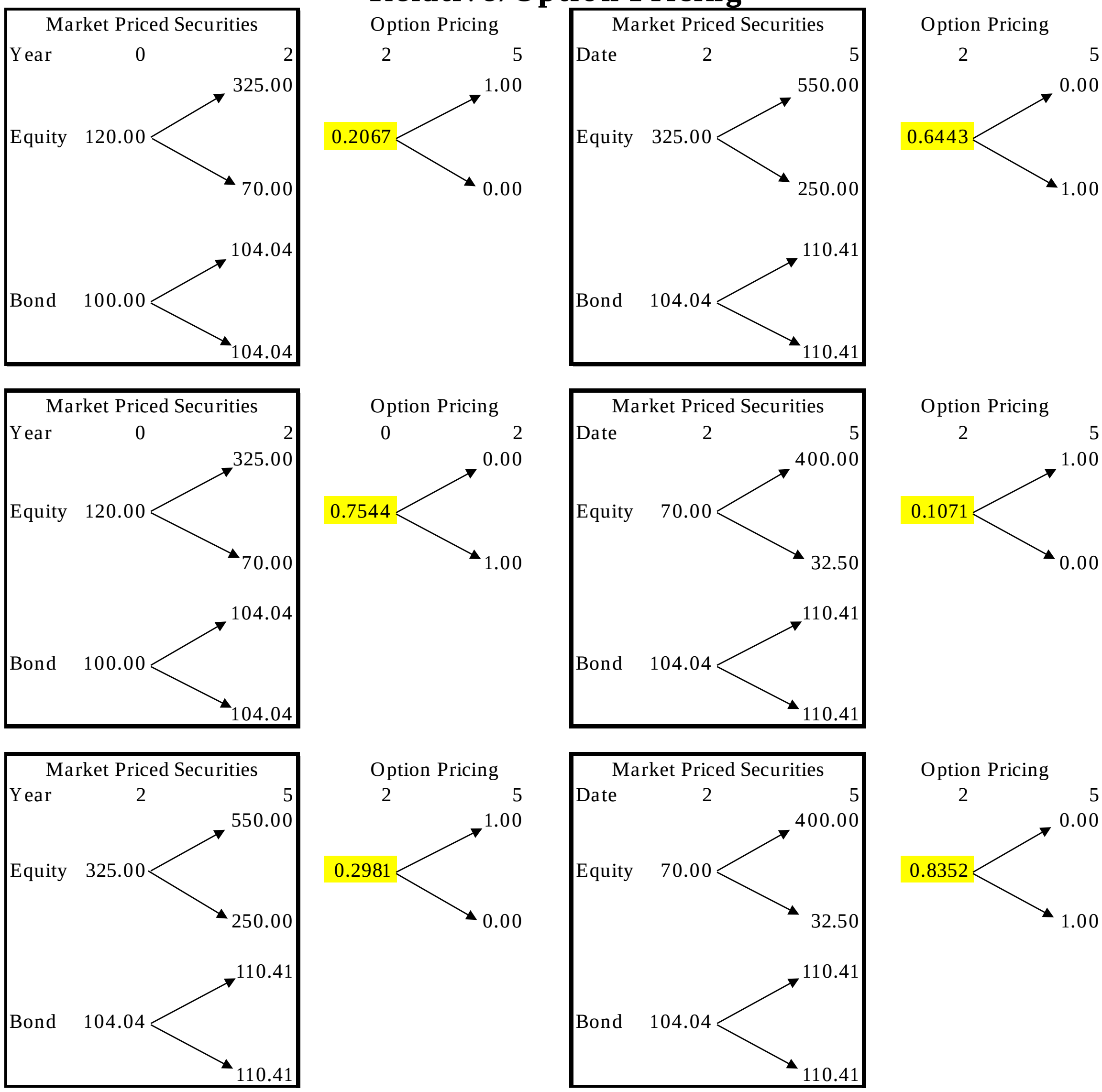


Table 7
Prices of Unit Payoffs

	Price at Year 0 of 1		Price at Year 2 of 1	Price at Year 0 of 1
Up-state	0.2067	Up-up-state	0.2981	0.0616
		Up-down-state	0.6443	0.1332
Down-state	0.7544	Down-up-state	0.1071	0.0808
		Down-down-state	0.8352	0.6301

Table 8
Payoffs and Present Values of Payoffs

	Equity	Preferred	Common	Options	Value per unit
Up-up-state	550.000	226.038	267.478	56.484	0.062
Up-down-state	250.000	119.635	126.711	3.654	0.133
Down-up-state	400.000	176.238	193.692	30.069	0.081
Down-down-state	32.500	32.500	0.000	0.000	0.630
Present value	**120.000**	**64.586**	**49.016**	**6.398**	0.906
OPM present values	**120.000**	**64.745**	**48.835**	**6.420**	