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by

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When SFAS 141R went into effect in 2009, it introduced the requirement that buyers report the fair value of contingent consideration at the acquisition date and, in the case of asset or liability classified contingent consideration, at each reporting date. Valuing contingent consideration can be difficult. This article explores those difficulties and proposes an approach that we have found effective.

Introduction

Buyers and sellers often agree to include an earn-out or contingent payment as part of the consideration in an acquisition. Such contingent consideration may help close the deal because they bridge differences in opinion held by buyers and sellers about the current and future values of the transaction. Where former owners remain as managers, contingent consideration may also serve as an incentive.

Two Types of Contingent Consideration with Different Risks

In valuing contingent consideration, it is important to distinguish between two types that have very different risk characteristics:

Event-related contingent consideration is tied to the achievement of business objectives, such as the success of a scientific test. This type of contingent consideration is exposed to unsystematic risk, risk that is unrelated to economy-wide risks. Unsystematic risk can be eliminated through diversification in large portfolios and thus does not command an expected rate-of-return risk premium.

Market-related contingent consideration is based on variables such as the performance of the acquired company's sales or earnings or the common stock of the acquiring company. This type of contingent consideration is exposed to systematic risk (β-risk), risk that is related to economy-wide risks. Systematic risk cannot be eliminated through diversification in large portfolios and thus commands an expected rate-of-return risk premium.

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For event-related contingent consideration where systematic risk is zero, valuing contingent consideration requires estimates of the cash flows that result from the event, the probability of the event, and an estimate of the counterparty credit risk. For example, suppose there is a 75% chance of achieving an objective in two years and the payment for achieving it is $10 million. The risk-free rate of interest is 1.5%, and the counterparty credit risk is 2%. The value of this contingent consideration is $7.00 million $[(75\%)(\$10.00)]/[(1+0.015+0.02)^2=\$7.00)]$. Management can typically provide estimates of the probabilities of achieving event-related business objectives. The appraiser must estimate the counterparty credit risk.

This is a much more straightforward valuation than that required for market-related contingent consideration, where estimating systematic risk and required rates of return is often very challenging. This article describes an option-pricing approach for market-related contingent consideration that addresses these challenges.

Basic Valuation

Contingent consideration does not generally lend itself to valuation using the market or cost approach. Therefore, either the income or the option-pricing approach is most frequently used. The income approach requires estimates of the potential future cash flows and their risks. The starting point for estimating cash flows is typically the deal model. As part of the due diligence process the buyer typically estimates the probability of the acquired company achieving business objectives. Similarly, deal models typically contain estimates of the financial metrics, such as projected sales, that determine contingent payments. This information can be supplemented by discussions with management and independent research to develop the information necessary to value the contingent consideration.

We illustrate the income approach and some challenges associated with implementing this approach with an example of valuing a business. The business currently has

Table 1
Valuation of a Firm with Five Years of Cash Flow and a Sale at the End of Five Years

	Year					
	0	1	2	3	4	5
Expected Value of Free Cash Flow		$23.00	$26.45	$30.42	$34.98	$40.23
Terminal Value						$321.82
Discount Factor		0.889	0.790	0.702	0.624	0.555
Present Value of Free Cash Flow		$20.44	$20.90	$21.36	$21.84	$22.32
Present Value of Terminal Value						$178.59
Total Present Value	$285.45					

$20.00 million of free cash flow that is expected to grow at 15.0% per year for five years and then to be stable. At that time the firm will be worth eight times free cash flow. The required rate of return (weighted average cost of capital or WACC), as determined by the systematic risk of the firm, is 12.5%. The firm is worth $285.45 million (all values rounded) as shown in Table 1.

Now consider the cash flow in the first year in more detail: The expected value of the free cash flow in the first year is $23.00 million. Its value today is $20.44 million. We represent the risk of the cash flow in terms of a probability distribution shown in Table 2. While we may not always explicitly model a distribution of future cash flows, such a distribution is implicit in our use of an expected value. Estimating the probability distribution and the expected value of the cash flow of a contingent consideration is one of the two crucial steps in valuing it.

Consider now a contingent consideration payment defined as 10.0% of the first-year cash flow. The expected value of the payment for the contingent consideration would be 10.0% of $23.00 million or $2.30 million. Likewise the fair value of the contingent consideration would be 10.0% of $20.44 million or $2.044 million because the contingent consideration is simply a proportionate ownership of the cash flow.

Consider an alternative, and more typical, contingent consideration defined as 10.0% of the cash flow above $10.53 million. The expected value of the cash flow above $10.53 million is $12.47 million as shown in Table 2. What would the fair value of that cash flow be today?

It may be tempting to discount the $12.47 million at 12.5%, the firm's WACC, on the grounds that the cash flows of the firm and the contingent consideration are quite similar. In fact, they are quite different. The contingent consideration cash flows are 1.84 times riskier than the total cash flow, 57.0% volatility versus 30.9%. We use the data in Table 2 to illustrate this issue and its solution. Unfortunately this solution applies only for the special case shown in Table 2, where there are two cash flow tranches and one is risk free. For most market-related contingent consideration it is much more challenging to identify the systematic risk and the discount rate required for valuation.

The cash flows range from $10.53 million to $45.80 million as shown under the column total cash. We have also partitioned the cash flows into two tranches. Tranche 1 is a constant, risk-free[1] value $10.53 million, and tranche 2 is the remainder of total cash. We have partitioned the cash flows in this way to illustrate why 12.5% is not the appropriate discount rate for tranche 2 and to derive the appropriate rate.

The contingent consideration cash flow is 10% of the tranche 2 cash flow. We know the total value of the two tranches is $20.44 million. We can calculate the value of the risk-free tranche 1 by discounting the $10.53 million at the risk-free rate. Therefore, we can infer the value of the second tranche and its required rate of return or discount rate.

In this example, the risk-free rate of interest is 2.0%. Therefore, the fair value of the first tranche is $10.53/1.02=$10.32 million. It follows that the fair value of the second tranche must be $20.44−$10.32=$10.12 million. We can calculate the expected rate of return on the contingent consideration from the expected value of the contingent consideration, $12.47, and its fair value, $10.12. The expected rate of return is 23.2% ($12.47/$10.12−1=23.2%).

We can draw on the seminal research of Modigliani and Miller (MM) to confirm this result, because tranche 2 is a levered derivative of total cash.[2] MM derived the relationship between the required rate of return on equity

[1]At various points in this paper we invoke the common research assumption that cash flows are risk free. We do so to advance the understanding of this particular subject, appreciating that nothing is literally risk free. In this context we assume that the financial condition of the counterparty makes this payment certain.

[2]Franco Modigliani and Merton H. Miller, ''The Cost of Capital, Corporation Finance and the Theory of Investment,'' *American Economic Review* (June 1958):261–297. Miller received the Nobel Prize in Economics for his seminal work in corporate finance. This article employs a no-arbitrage argument and assumes the ability to take long and short positions in all assets.

Table 2
Valuing a Contingent Consideration in Two Tranches

Probability	Total Cash	Contingent Consideration Threshold (Tranche 1)	Contingent Consideration Basis (Tranche 2)
2%	\$45.80	\$10.53	\$35.26
9%	\$35.85	\$10.53	\$25.31
23%	\$28.06	\$10.53	\$17.53
32%	\$21.96	\$10.53	\$11.43
23%	\$17.19	\$10.53	\$6.66
9%	\$13.46	\$10.53	\$2.92
2%	\$10.53	\$10.53	\$0.00
100%			
Expected Value	\$23.00	\$10.53	\$12.47
Required Rate of Return	12.50%	2.00%	
Fair Value	\$20.44	\$10.32	\$10.12

in a capital structure with risk-free debt and equity. They showed that the required rate of return on equity depended on the required rate of return on the firm's assets, the risk-free rate of return, and the firm's debt-equity ratio. In our example, the total cash flows are the return to assets. The fixed payment of \$10.53 million, what we have called tranche 1, is the return to debt, and its value is \$10.33 million ($D=10.33$). The second tranche, the cash flows above \$10.53 million, is the return to equity and its value is \$10.12 million ($S=10.12$). MM derived an equation defining the required rate of return on equity $\overline{r_S}$ in terms of the required rate-of-return assets $\overline{r_A}$, the risk-free rate r, and the debt equity ratio, D/S:

$$\overline{r_S}=\overline{r_A}+\frac{D}{S}(\overline{r_A}-r),$$

$$\overline{r_S}=12.5\%+\frac{10.33}{10.12}(12.5\%-2.0\%)$$

$$=12.5\%+1.021(10.5\%)=23.2\%.$$

These two approaches determine the required rate of return in our illustration, but only for a division of the cash flows into a risk-free (debt) stream and a single risky (equity) stream. Contingent consideration payments rarely meet that requirement. For example, suppose the contingent consideration is 10% of the cash flow above \$15.00 million and zero otherwise. We can create a risk-free stream of \$15.00 and the basis for the contingent consideration, tranche 2. Table 3 displays these cash flows. The total cash flow is equal to the sum of the three tranches. We know the value of tranche 1 but not of tranches 2 and 3. We cannot infer their values by equating the value of total cash to the sum of the values of the three tranches because we have one equation and two unknown values. Likewise, we do not have the debt/equity division necessary to apply the MM approach.

It is informative to plot the contingent consideration, tranche 2, against the total cash flows. Figure 1 illustrates that the contingent consideration has a payoff that we can describe as a call option on the first year's cash flow with an exercise price of \$15.00 million. Recognizing that this contingent consideration is a call option emphasizes the challenge associated with its valuation. Before the development of the Black-Scholes-Merton (BSM) model for valuing options, researchers had very good approaches to estimating expected cash flows, but what stymied their efforts to value the options was an inability to estimate the appropriate discount rate.

Valuing Contingent Considerations as Real Options

We use the term ''real option'' to mean a contingent payoff where the underlying asset is not an asset priced in the market. Sales and earnings before interest and taxes (EBIT) are examples of what we call real assets. Common stocks, bonds, and commodities are examples of financial assets priced in the market.

In the current context, the crucial difference between a real asset and an asset priced in the market is that the expected growth rate of the former can take on any value and any pattern over time; the growth rate of the latter must be consistent with its risk and expected rate of return at all times. For example, the expected growth rate in sales of a firm can be 2.0%, 5.0%, 15.0%, or 50.0%. A real asset's expected growth rate can also be 25.0% for three years and then 1.0% for ten years. The expected growth rate of a financial asset priced in the market must be consistent with market conditions and its systematic risk. Put another way, traded prices incorporate expectations of the future so that their expected growth rates are consistent with the risk-free rate of interest, the market price of systematic risk, and the traded asset's systematic risk.

Table 3
Valuing a Contingent Consideration in Three Tranches

Probability	Total Cash	Contingent Consideration Threshold (Tranche 1)	Contingent Consideration Basis (Tranche 2)	(Tranche 3)
2%	$45.80	$15.00	$30.80	$0.00
9%	$35.85	$15.00	$20.85	$0.00
23%	$28.06	$15.00	$13.06	$0.00
32%	$21.96	$15.00	$6.96	$0.00
23%	$17.19	$15.00	$2.19	$0.00
9%	$13.46	$15.00	$0.00	($1.54)
2%	$10.53	$15.00	$0.00	($4.47)
100%				
Expected Value	$23.00	$15.00	$8.23	($0.23)
Required Rate of Return	12.50%	2.00%	?	?
Fair Value	$20.44	$14.71	?	?

Notwithstanding the differences between real assets and financial assets, we are able to value real options quite readily using well-known option-pricing methods. Doing so requires a relatively simply modification of the standard option-pricing formulas similar to the modification to accommodate dividends paid continuously. Without dividends, the BSM analysis produced pricing formulas in which the only rate of return is the risk-free rate of return r. When a stock pays a continuous dividend rate d, the rate of return in the BSM formula is reduced by that dividend rate and becomes $(r - d)$. In the Appendix we develop a similar result for options on real assets: The rate of return in the real asset option-pricing formulas is $(r + g)$, where g is the real asset's growth rate minus the required rate of return appropriate for the real asset:[3]

$$c = Se^{gt}N(d_1) - Xe^{-rT}N(d_2) \text{ and}$$

$$p = Xe^{-rT}N(-d_2) - Se^{gT}N(-d_1),$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + g + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}.$$

These equations are structurally identical to those derived for the case of continuous dividends with the growth rate taking the place of the dividend rate and the minus sign reversed to a plus sign.

We illustrate these results with an example in which we value two contingent claims based on sales. One contingent consideration is the value of sales above $10.53 million in one year, and the other is the value in one year above $15.00 million. The current level of sales is $20.00 million, and sales are expected to grow 15.0% in the next year and to have a volatility of 30%. The risk-free rate of return is 2.0%, the market risk premium is 7.0%, and the sales beta is 1.50. Therefore the required rate of return on sales $\overline{r_S}$ is 12.5% [2.0%+1.5(7.0%)] and g is 2.5% (15.0%−12.5%).

We display the values of these two contingent considerations in Table 4 under the columns Option 2 and 3. For comparison purposes, we include as option 1 a call option on a market-priced security with the same parameters as option 2.[4] Because we wanted to link these results to the examples discussed earlier, we selected parameters for the real options that are the same as those used in Tables 2 and 3. The probability distribution in Table 2 is a discrete approximation of a lognormal distribution with a volatility of 30%. Consequently the value of option 2 in Table 4, $10.14, is approximately equal to the value of the option shown in Table 3. Similarly option 3 is the continuous distribution equivalent of the contingent consideration described in Table 3 as tranche 2. Taking its value, $6.09, as a good approximation of the value of the value of tranche 2 in Table 3, allows us to estimate the required rate of return on tranches 2 and 3, 35.0% (8.23/6.09−1=35.0%) and −35.8% (−0.23/(20.44−14.71−6.09)−1=−35.8%), respectively. These values emphasize how difficult it is to anticipate the size of the required rates of returns on option-like cash flows.

Required Rates of Return on Options

Although our primary interest is in the valuation of market-related contingent consideration using option pricing, we will make one more set of observations about

[3]In the following equations, S, X, c, and p are the values of the underlying asset, the strike price, a call option, and a put option, respectively. T is the time to expiration of the option, and σ is the volatility of the rate of return on the underlying asset. $N(z)$ is the area under a unit normal probability distribution from minus infinity to z.

[4]Consistent with the BSM equation, we converted all rates to their continuously compounded equivalents.

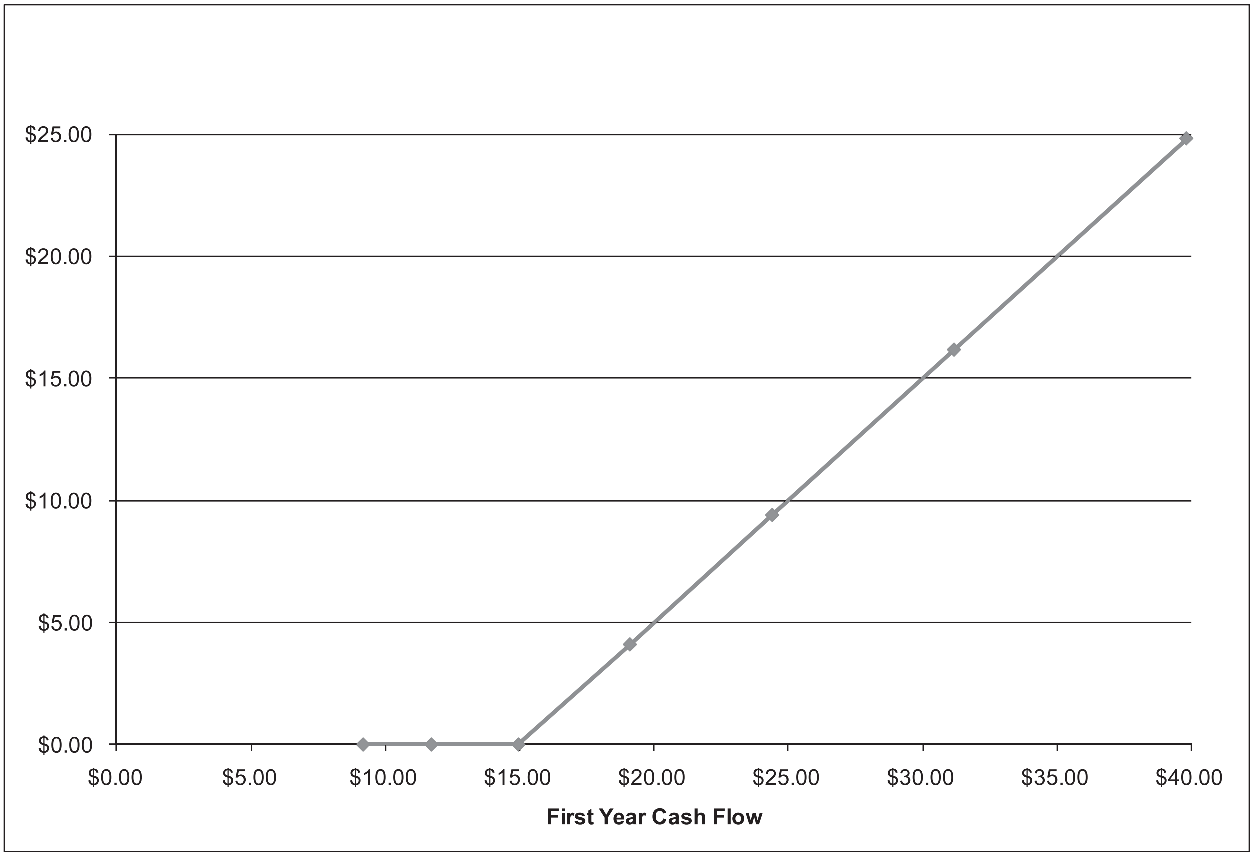


Figure 1
First-Year Cash Flow above the Contingent Consideration Threshold (millions)

the required rates of return on options. Rubinstein provides an analytic solution for the required rate of return on call options.[5] Using the parameters from Tables 1 and 2, we have displayed the results of his analysis in graphical form in Figure 2.

For deep-in-the-money call options the required rate of return approaches the required rate of return on the underlying asset, in this case 12.5%. For a call with a strike $10.00, which is halfway between completely in-the-money and at-the-money, the required rate of return is 23.0%. For an at-the-money call, a strike of $20.00, the required rate of return is 55.0%. For a call with a strike of $30.00 the required rate of return is 106.0%. These results emphasize one of the strongest arguments for valuing market-related contingent consideration using a real options approach: the difficulty of identifying the appropriate discount rate to use for contingent considerations with option-like payoffs. The next section discusses this and other issues related to effective valuation of contingent consideration.

[5]Mark Rubinstein, ''A Simple Formula for the Expected Rate of Return on an Option over a Finite Holding Period,'' *Journal of Finance* 39 (1984):1503–1509.

Valuation Challenges and Solutions

Numerical methods provide additional support for the real options approach to valuing contingent considerations. While there will undoubtedly be situations in which the closed-form solutions will be useful tools for valuing contingent consideration, many cases will call for numerical methods. The results derived in the Appendix can be applied to both of the most popular numerical methods, lattices, and Monte Carlo simulation.

Table 4
Valuing a Sales-Based Contingent Consideration

	Market-Priced Asset	Real Asset	
	Option 1	Option 2	Option 3
S	$20.00	$20.00	$20.00
X	$10.53	$10.53	$15.00
r	1.98%	1.98%	1.98%
s		11.78%	11.78%
g		2.20%	2.20%
σ	30.0%	30.0%	30.0%
T	1.00	1.00	1.00
Call Value	$9.70	$10.14	$6.09

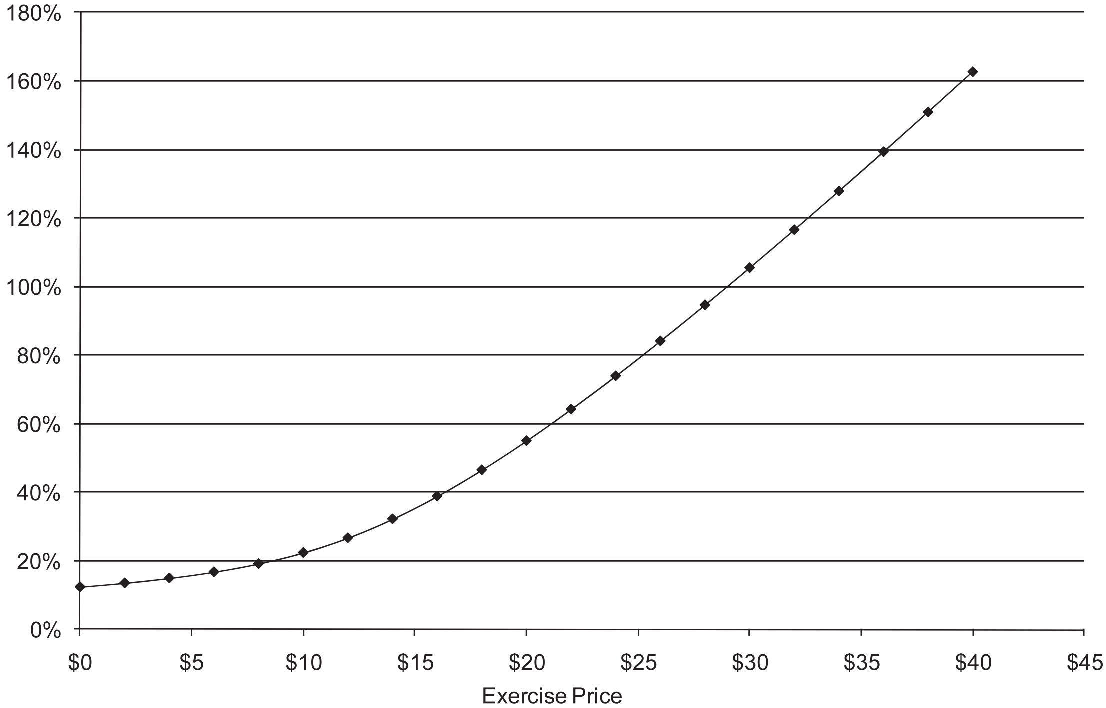


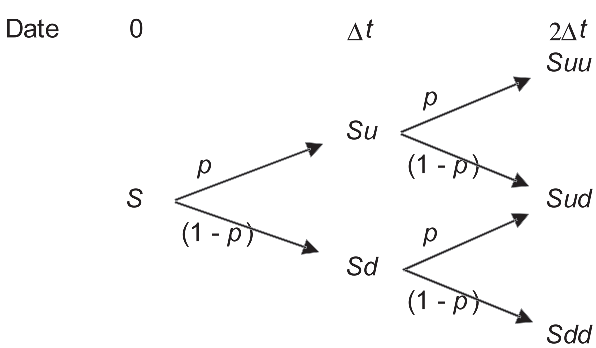
Figure 2

Required Rates of Return on Call Options for Different Strike Prices: Stock Price=$20, Holding Period=1 Year, Risk-Free Rate=2%, Volatility=30%

With respect to lattices, we consider only the case in which g is constant through time. In that simple case, the modified Cox-Ross-Rubinstein equations for the construction of a recombining lattice are

$$u=e^{\sigma\sqrt{\Delta t}},\ d=\frac{1}{u},\text{ and } p=\frac{e^{(r+g)\Delta t}-d}{u-d},$$

where u and d are the factors that determine the upward and downward movement of the asset and p and $(1-p)$ are the probabilities of upward and downward movements in a lattice of the form.



For the Monte Carlo simulation implementation, we explicitly acknowledge the possibility of time-varying parameters. The evolution of the asset price is

$$S_{t+\Delta t}=S_t e^{\left(r_t+g_t-\frac{\sigma_t^2}{2}\right)\Delta t+\sigma z\sqrt{\Delta t}},$$

where z is a unit normal random variable. Monte Carlo simulation is likely to be especially useful because many contingent considerations are path dependent. For example, the contingent consideration could be (1) defined by average sales, (2) depend on periodic growth rates, and (3) could include multiple payments that depend on previous payments. We will illustrate these cases in the following section, which includes examples, but first we will consider challenges to implementing the real options approach.

Challenges in Adopting the Real Options Approach

There are specific situations in which the real options approach is a very useful. By no means, however, do we want to suggest that it is the only reasonable approach. Other income approaches, for example, the probability weighted expected return method, can also be effective. The option-pricing method is likely to be most valuable for cases in which the value of the contingent consideration is relatively large and it is paid over a relatively long term, the option is not deep in the money, and the systematic risk is not close to zero. Deciding to apply the real options approach to value contingent consideration involves conceptual judgment and empirical challenges.

The conceptual issue is whether valuing a contingent consideration *as if* the BSM assumptions hold provides insight into its fair value. ASC Topic 820 defines fair value as ''the price that would be paid to receive an asset or paid to transfer a liability in an orderly transaction between market participants at the valuation date.'' Contingent consideration and their underlying assets are often not traded, and any valuation will often be based on unobservable inputs. In this author's opinion, the pricing of common stock in private companies and the pricing of employee stock options provides precedence for using a real options approach to valuation.

Chapter 10, ''Valuation of Preferred versus Common Stock'' in the AICPA practice guide *Valuing Equity* endorses the BSM equation as one of the acceptable ways to value the components of complex capital structures of private companies. It is also typical to use BSM to price options on private company stock even though neither the option nor the underlying stock is traded. One can argue that common stock in a private company is more similar to the securities envisioned by the BSM model than contingent consideration, but that does not mean that the BSM does not provide meaningful estimates of the fair value of contingent consideration.

The empirical challenges involve estimating the volatility and systematic risk of the real assets underlying the contingent consideration. We illustrate a direct and an indirect approach to estimating these parameters for two common underlying assets: EBIT and revenues.

The direct approach to estimating volatility and systematic risk employs market rate-of-return data and quarterly financial data for the ten-year period 2001 to 2010. The sample is 122 of the largest nonfinancial U.S. public companies. For each company, we computed equity betas using company and market rates of return in regressions. For EBIT and revenues we also computed betas using quarterly growth rates for individual companies as the dependent variable and the market rate of return as the independent variable. Table 5 summarizes the results.

The average equity beta is 1.00, suggesting that the sample is representative of companies in the market. The standard deviation of the equity betas is 0.27, and they range from 0.50 to 1.72. The average EBIT beta is 0.16. The standard deviation of the EBIT betas is 0.82, and they range from −3.71 to 2.76. The average value is statistically significantly different from zero, but 41% of all of the EBIT betas are negative. The average sales beta is 0.05. The standard deviation of the sales betas is 0.26, and they range from −0.99 to 1.20. The average sales beta is statistically significantly different from zero, but 40% of all of the sales betas are negative.

One interpretation of these results is that sales and earnings of large companies have little if any systematic risk. While the average values are statistically different from zero, only 6% of the individual company betas are statistically significantly different from zero. One can argue that what we are observing is random variations around true values that are zero or very close to zero. A second interpretation is that the data and methods used to estimate the betas are inadequate to identify the true systematic risk and that more effective econometric methods and/or better data may produce different results. To date, however, these are the best results we have produced. We think one of the potential sources of problems is that we are regressing changes in the values of flow variables, EBIT and sales, on changes in the value of a stock variable, the market index.

We have a strong a priori belief that the EBIT and sales of large companies do have positive systematic risk, and therefore we investigate an indirect approach to estimating these betas. We estimate proxies for EBIT and sales betas based on a firm's equity beta, estimated using the regression analysis as previously described. We think the equity beta is a good proxy for a firm's net income beta because net income is a measure of equity's claim on the firm. Similarly, we think that a firm's asset beta is a good proxy for its EBIT beta. There are standard approaches to calculate a firm's asset beta from its equity beta by adjusting for the financial leverage effect of debt. We employ a method that Hamada proposed.[6] The beta of assets β_A is the beta of equity β_E adjusted for the debt-to-equity ratio (D/E) and a tax rate t effect:

$$\beta_A = \frac{\beta_E}{\left[1+(1-t)\frac{D}{E}\right]}.$$

Although less familiar, there is a similar way to calculate a firm's sales beta from its EBIT beta by adjusting for operating leverage. Brealy, Myers, and Allen show that the beta of sales or revenue β_R is equal to the beta of assets adjusted for operating leverage in the form of the ratio of the present value of fixed costs PV_F to the present value of assets PV_A:[7]

[6] R. S. Hamada, ''The Effect of the Firm's Capital Structure on the Systematic Risk of Common Stocks,'' *Journal of Finance* 27 (1972):435–452.

[7] Richard Brealy, Stewart Myers, and Franklin Allen, *Principles of Corporate Finance,* 8th edition (New York: McGraw Hill Irwin, 2006), 225–226.

Table 5
Estimating Betas and Volatilities Directly from Regression Analysis

	Equity		EBIT		Sales	
	Beta	Volatility	Beta	Volatility	Beta	Volatility
Average	1.00	25%	0.16	61%	0.05	25%
Standard Deviation	0.27	7%	0.82	40%	0.26	14%
Minimum	0.50	12%	−3.71	9%	−0.99	3%
Maximum	1.72	49%	2.76	255%	1.20	83%
Percent Less than 0.00	0.00%		41%		40%	

$$\beta_R = \frac{\beta_A}{\left[1 + \frac{PV_F}{PV_A}\right]}.$$

Table 6 reports the results of our empirical analysis of the 122 large nonfinancial U.S. companies using these adjustments to calculate asset and sales betas and volatilities. We estimated the ratio of PV_F to PV_A based on the capitalized relative values of fixed costs and EBIT over the period 2001 to 2011, with the former capitalized at a bond rate and the latter capitalized at the cost of capital. The statistics for the equity betas are the same as those in Table 5. The average EBIT beta is lower, 0.81, because of the removal of the financial leverage effect on beta. The distribution of these estimates of the EBIT betas is very different from that shown in Table 5. The standard deviation of the EBIT betas is 0.23, and there are no negative values in the range from 0.39 to 1.36. The average sales beta is 0.33, and there are no negative values in the range from 0.09 to 1.13. We believe that these results are more consistent and plausible estimates of the systematic risk of EBIT and sales.

While we think these results are informative, there are significant challenges in implementing operational de-leveraging. In general, it is not easy to parse costs into fixed and variable categories. In particular, contingent considerations can have very short and very long terms, and what is a fixed cost in the short term can be a variable cost in the longer term.[8] As a starting point, we think of cost of sales as the key variable cost and of selling, general, and administrative costs as the key fixed cost. Although there is no theoretically pure solution to this problem, we have found, in practice, that we can identify plausible divisions of costs that produce reasonable estimates of systematic risk for sales. In many cases, it is also true that the value of the contingent consideration is not especially sensitive to alternative divisions of fixed and variable costs.

[8] We have valued some contingent considerations with one-year terms and one with an 18-year term.

Based on the statistical results and our experience with private companies and project specific analyses, we believe that the use of proxies to estimate EBIT and sales betas and volatilities holds considerable promise in the application of the real options approach to contingent valuation. The results are consistent across companies and easily replicated.

Examples

We provide two illustrative contingent consideration valuation examples.

Application of the modified BSM call option formula

The contingent consideration is determined by the level of sales in the second year. The current level of sales is \$100 million, and sales are expected to grow at 22.0% per year for the next two years. The volatility of the sales rate is 30.0%, and the risk-free rate is 2.0%.

The contingent consideration has two parts: (1) a fixed payment of \$5 million, if the second-year sales exceed \$200 million, and (2) a variable payment equal to 20.0% of sales above \$200 million. We value the contingent consideration for two firms. Firm A has a sales beta of 0.0, and Firm B has a sales beta of 0.5. The market risk premium is 7.0%. Table 7 provides the values of the contingent consideration and illustrates their calculation.

The values of the \$5 million payments are the expected present value of \$5.0 million. The probability of the payment is equal to $N(d_2)$, the ''risk-neutral'' probability of sales exceeding \$200 million. The discount factor is $e^{-(1.98\%)(2)}$. The values of the payments tied to sales above \$200 million are based on the modified BSM formula implemented in Table 4. The systematic risk of sales of Firm B decreases the values of both types of contingent consideration payments.

Monte Carlo simulation

The second example involves a hypothetical deal in which the purchaser of a company has retained the owner/manager and the manager's team to manage the acquired company. The purchaser will pay an earn-out based on

Table 6
Estimating EBIT Betas and Volatilities Indirectly from Equity Estimates

	Equity		EBIT		Sales	
	Beta	Volatility	Beta	Volatility	Beta	Volatility
Average	1.00	25%	0.81	20%	0.33	8%
Standard Deviation	0.27	7%	0.23	6%	0.24	6%
Minimum	0.50	12%	0.39	9%	0.09	2%
Maximum	1.72	49%	1.36	39%	1.13	30%
Percent Less than 0.00	0.00%		0.00%		0.00%	

sales achieved over the next three years. Specifically, the purchaser will pay a $5 million earn-out if (1) sales in the third year exceed $30 million and (2) aggregate sales over the first three years exceed $85 million.

Based on the deal model the current level of sales is $20 million, and the anticipated growth rates for sales for the next three years are 15.0%, 15.0%, and 10.0%. A comparison of publicly traded comparable companies suggests a sales volatility of 20.0% and a sales beta of 0.10. The risk-free rate is 2.0%, and the market risk premium is 7.0%. To value this contingent consideration we conduct a Monte Carlo simulation of annual sales S:

$$S_{t+\Delta t} = S_t e^{\left(r + g_t - \frac{\sigma^2}{2}\right)\Delta t + \sigma z_i \sqrt{\Delta t}}.$$

The value of g for each year is the annual growth rate in sales reduced by the market-required rate of return for sales, which is 2.7%, the risk-free rate of return, plus the product of the market risk premium and the beta of sales.

Table 7
Valuing a Sales-Based Contingent Consideration

	Firm A	Firm B
S	$100.00	$100.00
X	$200.00	$200.00
R	1.98%	1.98%
s	1.98%	5.36%
G	17.90%	14.52%
Σ	30.0%	30.0%
T	2.00	2.00
d_1	−0.484	−0.644
d_2	−0.909	−1.068
Call Value	$9.99	$7.30
Value of 10% of Sales above $200 Million	$2.00	$1.46
$N(d_2)$	0.182	0.143
Value of $5 Million Fixed Payment	$0.87	$0.69
Beta	0.00	0.50
Market Risk Premium	6.77%	6.77%
Sales Risk Premium	0.00%	3.38%
Growth Rate of Sales	19.89%	19.89%

Table 8 displays the inputs for the Monte Carlo simulation and illustrates the simulation with fifteen iterations. In this sample of fifteen iterations, six satisfy the condition for third-year sales and five satisfy the condition for aggregate sales, but only four satisfied both conditions. Subject to the usual caveat that the model inputs are reasonable representations of reality, we believe that this is an excellent example of how the option-pricing approach can successfully measure value drivers that would be difficult to measure with accuracy in any other way.

For the entire sample 27.4% of the iterations satisfied both of the criteria for payment. The expected present value of the contingent consideration is $1.29 million [$5.0 (27.4%) $e^{-(1.98\%)(3)}$].

Conclusion

When the risk of contingent consideration is not market related, it is relatively straightforward to value the contingent consideration using the income approach. The valuation requires estimates of the expected cash flows of the contingent consideration and a discount rate composed of the risk-free rate and a credit spread to reflect counterparty risk. Market-related contingent consideration is much more difficult to value, because it often possesses option-like features. There is no effective way to value options using standard discounted cash flow methods. Therefore, many market-related contingent considerations can be most effectively valued using option-pricing methods.

The option-pricing methods are already widely used to value nontraded financial instruments, for example, employee stock options and complicated incentive compensation, embedded derivatives, and complex capital structures. It is flexible and adaptable. It can be applied to simple payments in the form of the Black-Scholes equation. More complex payments can be valued using numerical methods. Monte Carlo simulation, in particular, is well suited to handle the interrelated financial metrics often found in contingent consideration.

Table 8
Monte Carlo Simulation Results

	Sales				
Iteration	Year 1	Year 2	Year 3	Total	Payment
1	$24.14	$24.60	$21.73	$70.47	$0.00
2	$17.86	$22.06	$22.72	$62.64	$0.00
3	$24.62	$27.62	$33.07	$85.31	$5.00
4	$23.66	$25.30	$34.49	$83.45	$0.00
5	$21.74	$24.64	$23.85	$70.23	$0.00
6	$22.32	$24.06	$28.96	$75.34	$0.00
7	$27.45	$30.85	$37.68	$95.98	$5.00
8	$17.83	$13.33	$12.68	$43.85	$0.00
9	$24.83	$33.35	$41.12	$99.31	$5.00
10	$26.93	$33.36	$27.10	$87.39	$0.00
11	$24.25	$19.85	$23.29	$67.39	$0.00
12	$20.96	$25.89	$32.18	$79.03	$0.00
13	$25.16	$35.60	$31.82	$92.58	$5.00
14	$19.88	$21.67	$21.41	$62.96	$0.00
15	$16.88	$19.24	$10.93	$47.05	$0.00

Appendix

Closed-Form Solution for the Values of Real Options

We make the standard option-pricing assumptions of lognormally distributed underlying assets, costless hedging, and the ability to trade in a risk-free asset. The analysis below is the traditional analysis that produces the partial differential equation that is the basis for the Black-Scholes call-and-put formulas, with one exception. Because real assets are not priced in the market place, their growth rates may be larger or small than the required rate of return on a financial asset with the same risk characteristics as the real asset. We represent the required rate of return on the real asset as μ and the amount by which the real asset's growth rate exceeds or falls short of μ as *g*. The real asset is *S*, and its volatility is σ. Options expire at date *T*. The diffusion process for the real asset is

$$dS = (\mu + g)Sdt + \sigma Sdz.$$

By Itô's lemma[9] the option price *f* diffusion process is

$$df = \left(\frac{\partial f}{\partial S}(\mu + g)S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial f}{\partial S}\sigma Sdz.$$

The discrete-time versions of these equations for a small interval of time Δt are

$$\Delta S = (\mu + g)S\Delta t + \sigma S\Delta z$$

and

$$\Delta f = \left(\frac{\partial f}{\partial S}(\mu + g)S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t + \frac{\partial f}{\partial S}\sigma S\Delta z.$$

Consider a security with price *V* with the same μ and σ as the real asset. This security is efficiently priced in the market. The diffusion process for this security is

$$dV = \mu Vdt + \sigma Vdz,$$

and in discrete time it is

$$\Delta V = \mu V\Delta t + \sigma V\Delta z.$$

Consider a portfolio composed of −1 unit of the derivative and $+\frac{\partial f}{\partial S}\frac{S}{V}$ shares of the priced security. This portfolio will cost

$$\Pi = -f + \frac{\partial f}{\partial S}\frac{S}{V}V = -f + \frac{\partial f}{\partial S}S.$$

The change in value of this portfolio over a small period of time Δt will be

$$\begin{aligned}\Delta\Pi &= -\Delta f + \frac{\partial f}{\partial S}\Delta S\\ &= -\left(\frac{\partial f}{\partial S}(\mu + g)S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t\\ &\quad - \frac{\partial f}{\partial S}\sigma S\Delta z + \frac{\partial f}{\partial S}(\mu S\Delta t + \sigma S\Delta z)\\ &= -\left(\frac{\partial f}{\partial S}gS + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t.\end{aligned}$$

[9]Itô's lemma is the stochastic calculus counterpart of the chain rule in ordinary calculus.

Because no stochastic term is found in this expression the rate of return on the portfolio must be the risk-free rate of return *r:*

$$\Delta\Pi = r\Pi\Delta t,$$

$$\left(\frac{\partial f}{\partial S}Sg + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)\Delta t = r\left(f - \frac{\partial f}{\partial S}S\right)\Delta t,$$

$$\left((r+g)\frac{\partial f}{\partial S}S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right) = rf,$$

Solving the equation subject to the European call-and-put option boundary conditions gives

$$f = \max(0, S_T - X) \quad \text{and} \quad f = \max(0, X - S_T),$$

and the solutions are the modified forms of the Black-Scholes call-and-put option formulas:

$$c = Se^{gT}N(d_1) - Xe^{-rT}N(d_2)$$

$$\text{and} \quad p = Xe^{-rT}N(-d_2) - Se^{gT}N(-d_1),$$

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + g + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

These equations should be familiar because they are structurally identical to those derived for the case of continuous dividends with the growth rate taking the place of the dividend rate and the minus sign reversed to a plus sign.[10]

Another intuitive way of thinking of this result is that the value of real-asset European options expiring at date *T* can be expressed in terms of the standard Black-Scholes-Merton formulas using the pseudo-price of the real asset. The pseudo-price *S** is the expected value of the real asset at date *T* discounted back to the present at the risk-adjusted discount rate μ:

$$S^* = E[S_T]e^{-\mu T} = Se^{(\mu+g)T}e^{-\mu T} = Se^{gT}$$

and

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + g + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{Se^{gT}}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln\left(\frac{S^*}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}.$$

[10]In the chapter ''Real Options'' in *Options Futures and Other Derivatives,* 6th ed. (Upper Saddle River N. J.: Prentice Hall, 2006), John Hull uses a risk-neutral pricing argument to develop the same conclusion. He concludes that the expected growth in a risk-neutral setting is the real-world growth rate, what we call $\mu + g$, minus the price of risk multiplied by volatility of the real asset. Adding subscripts *i* and *m* to indicate the real asset and the market, the connection between the two conclusions is expressed as

$$(\mu_i + g_i) - \lambda\sigma_i = (\mu_i + g_i) - \frac{\rho_{i,m}}{\sigma_m}(\mu_m - r)\sigma_i = (\mu_i + g_i) - \beta_i(\mu_m r) = r + g_i.$$