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In this article, we describe another method for calculating different discounts for lack of marketability (DLOMs) for each security in the capital structure based on its unique volatility. The primary merit of this method is that it requires a minimal change in appraiser practice. We also provide support for the use of incremental DLOMs when valuing securities based on a transaction in a non-marketable security.

Introduction

A contingent claims analysis (CCA) is one recommended way[1] of valuing non-marketable securities in a complex capital structure. In a CCA, individual securities are valued as combinations of Black-Scholes-Merton (BSM) call option spreads written on an underlying asset. It is relatively common practice to adjust the values indicated by the CCA of these securities to reflect their lack of marketability. Protective put models are often used to calculate discounts for lack of marketability (DLOMs), typically as the value of an at-the-money put written on the individual securities.[2] The rationale for applying the protective put model is that the put value measures the insurance cost of creating a floor value equal to the current value of an illiquid security. Because the protective put leaves the security holder with the upside above the floor, this insurance cost overstates the DLOM.[3] The hedging implicit in the put model is not typically feasible, and that may cause the model to understate the DLOM. There is a rich empirical literature on DLOMs based on, typically, the differences in transaction prices between traded securities and their corresponding 144A-restricted securities. Appraisers can use these results, the indications of the put models, and their judgment to conclude the appropriate values for DLOMs. Very often, however, appraisers default to the results of the put models.

In a recent article,[4] Stillian Ghaidarov noted that the put models introduce a theoretical inconsistency: The CCA assumes the value being allocated among the securities is log-normally distributed, and the protective put model assumes the value of the security is log-normally distributed. Both cannot be true. Ghaidarov's solution to this inconsistency is to measure the DLOM in terms of one or more put(s) written on the underlying asset. In addition to eliminating the theoretical inconsistency Ghaidarov identified, this approach calculates a different DLOM for each security.

In this article, we describe a method for calculating DLOMs that differ based on a security's unique volatility. The primary merit of this approach is that it requires a minimal change in appraiser practice. This approach does perpetuate the inconsistency that Ghaidarov identified. We argue that inconsistency is of little import. Appraiser's frequently treat both assets and equity as lognormally distributed, depending on the circumstances. For example, it is not uncommon to model assets or total equity as lognormally distributed in order to value common stock and, simultaneously, to value employee stock options using a BSM formula—that is, to treat common stock as also lognormally distributed. Moreover, many assumptions underlying the model are clearly not satisfied, in particular the assumption that perfect, costless hedging transactions are feasible. Lastly, we use call options to value the securities, but when we consider discounts for lack of marketability, we pretend that call options do not exist but that put options do. If we allowed both call and put options when considering DLOMs, we would conclude there is no such thing, because we would combine calls and puts to create forward sales. The point is a simple one: We do not judge our models based on their assumptions but rather on their performance.

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[1]American Institute of Public Accountants (2013) *Valuation of Privately-Held-Company Equity Securities Issued as Compensation*. AICPA Accounting and Valuation Guide 2013.
[2]See for example, Chaffe (1993) and Finnerty (2010). In this article we consider only the plain vanilla put method.
[3]Put models depend crucially on estimates of volatility, typically drawn from publicly traded comparable companies; whether these companies accurately represent the volatility of a private company, especially its event risk, is problematic.
[4]Ghaidarov (2009).

Table 1
Ghaidarov's CCA Example[5]

S	$50,000,000	$50,000,000	$50,000,000	
X	$0	$35,000,000	$175,000,000	
r	4.88%	4.88%	4.88%	
σ	50.0%	50.0%	50.0%	
T	2.000	2.000	2.000	
c	$50,000,000	$22,537,208	$1,290,568	
		Ownership claim		
Preferred	100%	0%	20%	
Common	0%	100%	80%	
		Value of ownership claim		Total
Preferred	$27,462,792	$0	$258,114	$27,720,905
Common	$0	$21,246,640	$1,032,454	$22,279,095

In the next section, we review Ghaidarov's initial example. In the following section we outline an alternative approach to estimating different DLOMs for securities with different volatilities. We show that the DLOMs produced by this model and those reported by Ghaidarov in his article are very similar. In the last section, we demonstrate how to calculate an incremental DLOM to determine the non-marketable value of securities in a complex capital structure when back-solving for an enterprise value based on the non-marketable value of a newly issued security. A summary concludes the article.

Ghaidarov's Initial Example

Ghaidarov uses a simple two-security capital structure in his 1st illustration of the recommended method. The underlying asset, *S*, is an equity pool worth $50 million. Preferred stock receives 100% of equity up to $35 million; common stock receives 100% of equity between $35 million and $175 million; preferred and common stock split equity 20:80 above $175 million. The other terms of the CCA are shown in Table 1.

The author notes that only *S* is lognormally distributed. Therefore, a protective put valued as a put on either the participating Preferred or the Common is not strictly appropriate because the prices of the preferred and the common stock are not lognormally distributed. Instead, he proposes that we write the put on the underlying asset, *S*. For the Preferred, this is a put on *S* with a strike price equal to the value of the Preferred. For the Common, the protective put is a combination of a long put and a short put. The former has an exercise price equal to the liquidation preference of the Preferred plus the value of Common. The latter has an exercise price equal to the liquidation preference of the Preferred. This method produces DLOMs of 7.6% and 48.5% for Preferred and Common, respectively, as shown in Table 2. The combined DLOM for the two securities is 25.8% of the total value. Figure 1 illustrates that the common stock plus the two puts produce a floor value of $22.3 million with appreciation potential when the equity pool is worth more than $57.3 million.[6]

Ghaidarov promotes his model by comparing its results to those produced by using asset volatility for all securities. This alternative produces identical DLOMs of 21.9% for both securities. Because the Preferred and Common have different DLOMs, a method that yields the same answer for each is clearly flawed. However, there are more appealing alternative approaches that Ghaidarov did not consider. One straightforward alternative is based on estimating different volatilities for each security in a CCA and different DLOMs. We think this approach is easier to implement.

Later in the article we show that the volatilities are 16.2% and 92.1% for preferred and common stock,[7] respectively, and the DLOMs are 4.8% and 41.7%, respectively. How should we think about two models where the DLOMs are 7.6% versus 4.8% and 41.7% versus 48.5%? These differences exist in a context where (1) the protective put is not a direct measure of a DLOM, (2) empirical support for any analytical results is extremely limited,[8] and (3) estimates of volatilities are

[5] *S* is the BEV, *X* is the breakpoint, *r* is the risk-free rate of interest, *q* is the dividend rate, *σ* is the volatility of *S*, *T* is the time to the liquidity event, and *c* is the value of the call option.

[6] The sum of the liquidation preference of the Preferred and the fair value of the Common is $57.3 million. In this example, as is usual in practice, the terms used in the CCA and in the calculation of the DLOM are identical. If a security can be effectively marketed prior to a liquidity event, then the DLOM model may have a shorter term than the CCA.

[7] We discuss the calculation of these volatilities below.

[8] Finnerty links his proposed method to empirical results and finds a direct relationship between a security's volatility and its DLOM.

Table 2
Ghaidarov's Protective Put Example[9]

	Preferred	Common	
	Protective Put	Protective Put	
S	$50,000,000	$50,000,000	$50,000,000
X	$27,720,905	$57,279,095	$35,000,000
r	4.88%	4.88%	4.88%
σ	50.0%	50.0%	50.0%
T	2.000	2.000	2.000
p	$2,117,346	$15,083,022	−$4,283,240
p/Fair Value	7.6%	67.7%	−19.2%
		$10,799,782	
		48.5%	

imprecise. Therefore, we believe these values are sufficiently close to Ghaidarov's values of 7.6% and 48.5% that it is impossible to argue that one result is more accurate than the other. We will show below why we believe our values are likely to be easier for appraisers to calculate. First, we describe in more detail this alternative approach to calculating differential DLOMs and compare its results to the full set of examples that Ghaidarov discussed.

CCA and Security-Specific DLOMs: An Alternative Approach

The alternative approach we are proposing is built around one of the very important relationships in option theory, an option's delta. In addition to being the basis for creating a risk-free hedge, the delta links the volatility of the underlying asset to the volatilities of derivative securities and, more importantly, portfolios of derivative securities written on the underlying asset. We can use this relationship to calculate the instantaneous volatility of each of security in a CCA. The computational process is very similar to the calculation of the value of the security and, thus, very straightforward. We will first derive the relationships and then illustrate the application and compare its results to those produced by Ghaidarov's approach.

The delta of a call option, Δc, measures the change in value of the call option relative to the change in value of the underlying asset[10]:

$$\Delta c = \frac{\partial c}{\partial S} = N(d_1)$$

The instantaneous volatility of a call option is the product of the delta of the call option, the leverage ratio, S/c, and the volatility of the underlying asset, σ_S[11]:

$$\sigma_c = N(d_1)\frac{S}{c}\sigma_S$$

In a CCA, the value of the underlying asset is partitioned into call spreads:

$$S = (c_0 - c_1) + (c_1 - c_2) + \ldots + (c_{n-1} - c_n) + c_n$$

The value of each security, L_i, is a weighted average of these call spreads, where $w_{i,k}$ is the proportion of call spread $(c_k - c_{k-1})$ security i claims:

$$L_i = w_{i,o}(c_0 - c_1) + w_{i,1}(c_1 - c_2) + \ldots w_{i,n-1}(c_{n-1} - c_n) + w_{i,n}c_n$$

If we take the partial derivative of L with respect to S, we have

$$\begin{aligned}\frac{\partial L_i}{\partial S} &= w_{i,0}\left(\frac{\partial c_0}{\partial S} - \frac{\partial c_1}{\partial S}\right) + w_{i,1}\left(\frac{\partial c_1}{\partial S} - \frac{\partial c_2}{\partial S}\right) \\ &\quad + \ldots + w_{i,n-1}\left(\frac{\partial c_{n-1}}{\partial S} - \frac{\partial c_n}{\partial S}\right) + w_{i,n}\frac{\partial c_n}{\partial S} \\ &= w_{i,0}\left(N(d_1)_0 - N(d_1)_1\right) \\ &\quad + w_{i,1}\left(N(d_1)_1 - N(d_1)_2\right) + \ldots \\ &\quad + w_{i,n-1}\left(N(d_1)_{n-1} - N(d_1)_n\right) + w_{i,n}N(d_1)_n \\ &= WND_i\end{aligned}$$

[9] *p* is the value of the put option; fair value is $27.7 million for the preferred stock and $22.3 million for the common stock.

[10] See Whaley (2006), chapter 12, especially pp. 439–444. $N(d_1)_k$ has the usual definition in a BSM formula, for delta spread *k*.

[11] I would like to thank my colleague Peter Geday for suggesting the inclusion of these calculations.

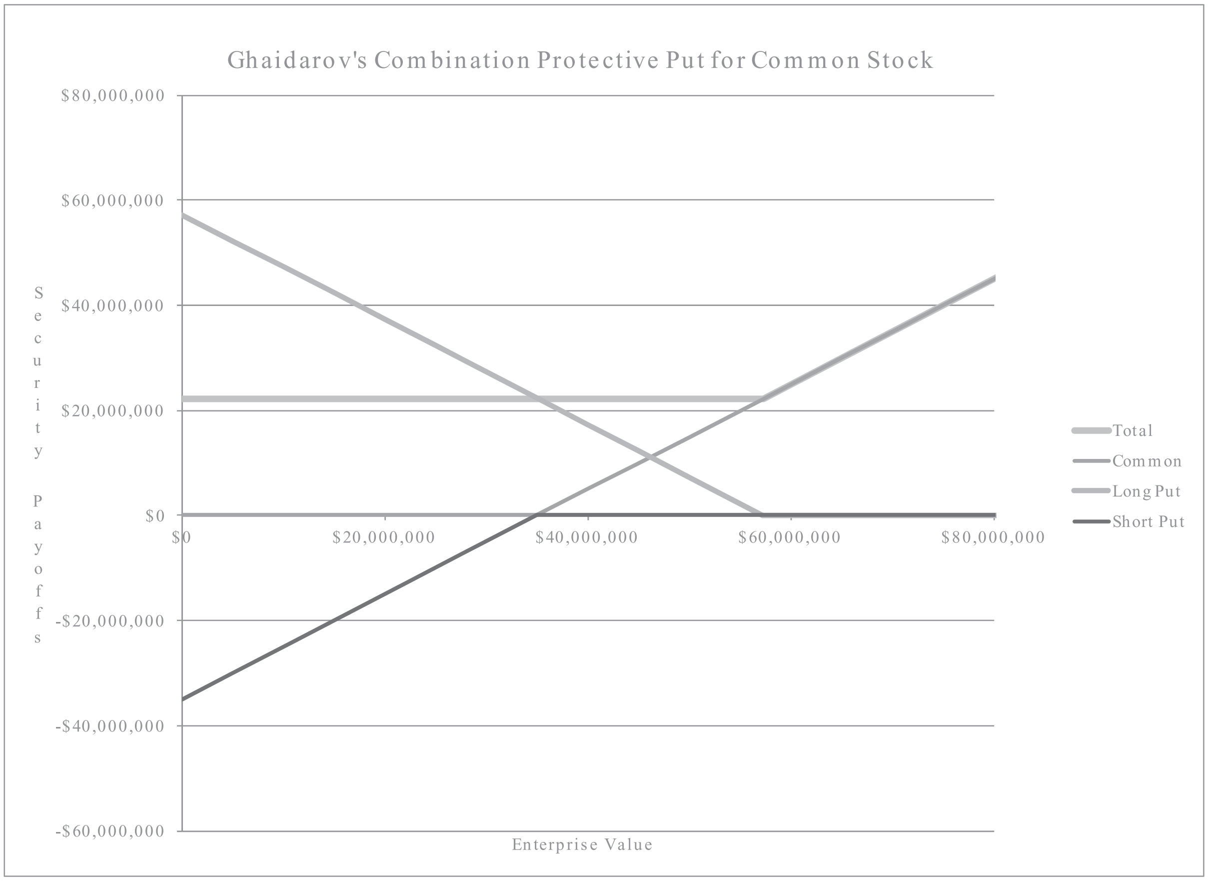

Figure 1
Ghaidarov's Combination Protective Put for Common Stock

Table 3
CCA Valuation and Calculation of Differential Volatilities

S	$50,000,000	$50,000,000	$50,000,000		
X	$0	$35,000,000	$175,000,000		
r	4.88%	4.88%	4.88%		
σ	50.0%	50.0%	50.0%		
T	2.000	2.000	2.000		
d_1	41.844	0.996	−1.280		
C	$50,000,000	$22,537,208	$1,290,568		
Call Spread	$27,462,792	$21,246,640	$1,290,568		
N(d_1)	1.000	0.840	0.100		
Delta Spreads	0.160	0.740	0.100		
		Ownership claim			
Preferred	100%	0%	20%		
Common	0%	100%	80%		
		Value of ownership claim		Total	
Preferred	$27,462,792	$0	$258,114	$27,720,905	
Common	$0	$21,246,640	$1,032,454	$22,279,095	
		Security delta		Total (*WND*)	Volatility
Preferred	0.160	0.000	0.020	0.180	16.2%
Common	0.000	0.740	0.080	0.820	92.1%

Table 4
Security-Specific Volatility

	Preferred	Common
	Protective Put	Protective Put
S	\$27,720,905	\$22,279,095
X	\$27,720,905	\$22,279,095
r	4.88%	4.88%
σ	16.2%	92.1%
T	2.000	2.000
p	\$1,336,878	\$9,289,997
p/Fair Value	4.8%	41.7%

The variable WND_i is the weighted average of the delta spreads for security *i* of the CCA. The volatility of each security L_i is

$$\sigma_{L_i} = WND_i \frac{S}{L_i} \sigma_S$$

When we calculate the value of a security in a CCA analysis, we multiply the values of call spreads by the percentage claim that a security has on that spread and then we sum them. That is exactly how we calculate *WND* for each security, except that we are multiplying delta spreads rather than call spreads. Table 3 illustrates this process for Ghaidarov's initial example.

We calculate the volatilities of the Preferred and the Common, securities 1 and 2, as

$$\sigma_1 = WND_1 \frac{S}{L_1} \sigma_S = 0.18 \frac{\$50.0}{\$27.7} 50\% = 16.2\% \text{ and } \sigma_2 = WND_2 \frac{S}{L_2} \sigma_S = 0.82 \frac{\$50.0}{\$22.3} 50\% = 92.1\%$$

We calculate the DLOMs in the usual way, as shown in Table 4. The DLOMs are 4.8% for the Preferred and 41.7% for the Common. These results compare with Ghaidarov's values of 7.6% and 48.5%, respectively. Given everything we know and do not know, there is no arguing precedence for one or the other of these two sets of estimates.

Ghaidarov considered a 2nd example. The convertible Preferred Series A and B each have a liquidation preference of \$2.0 million and have equal seniority. Series A is convertible to Common at \$1.00 per share. Series B is convertible at \$2.00 per share. There are 3.0 million shares of common stock. Table 5A reports the results of the CCA allocation and the computation of the security-specific volatilities. Table 5B reports the calculation of the DLOMs and compares them with Ghaidarov's results. While the two sets of DLOMs for Series A,

Table 5A
Contingent Claims Analysis

S	\$5,000,000	\$5,000,000	\$5,000,000	\$5,000,000		
X	\$0	\$4,000,000	\$7,000,000	\$12,000,000		
r	4.88%	4.88%	4.88%	4.88%		
σ	50.0%	50.0%	50.0%	50.0%		
T	2.000	2.000	2.000	2.000		
d_1	38.587	0.807	0.016	−0.747		
c	\$5,000,000	\$1,992,429	\$977,951	\$343,555		
Call Spread	\$3,007,571	\$1,014,478	\$634,396	\$343,555		
N(d_1)	1.000	0.790	0.506	0.228		
Delta Spreads	0.210	0.284	0.279	0.228		
		Ownership claim				
Series A	50%	0%	40%	33%		
Series B	50%	0%	0%	17%		
Common	0%	100%	60%	50%		
		Value of ownership claim			Total	
Series A	\$1,503,786	\$0	\$253,759	\$114,518	\$1,872,062	
Series B	\$1,503,786	\$0	\$0	\$57,259	\$1,561,045	
Common	\$0	\$1,014,478	\$380,638	\$171,777	\$1,566,893	
		Security delta			Total	Volatility
Series A	0.105	0.000	0.111	0.076	0.292	39.0%
Series B	0.105	0.000	0.000	0.038	0.143	22.9%
Common	0.000	0.284	0.167	0.114	0.565	90.1%

Table 5B
Calculation of DLOMs Using Protective Puts

	Protective Puts			
	Series A	Series B	Common	
S	$1,872,062	$1,561,045	$1,566,893	
X	$1,872,062	$1,561,045	$1,566,893	
r	4.88%	4.88%	4.88%	
σ	39.0%	22.9%	90.1%	
T	2.000	2.000	2.000	Total
Put	$306,957	$127,263	$640,304	$1,074,523
Put/Fair Value	16.4%	8.2%	40.9%	21.5%
Ghaidarov	13.8%	9.8%	50.6%	24.1%

Series B, and Common—16.4%, 8.2%, and 40.9% and 13.8%, 9.8%, and 50.6%—again differ, they are similar in relative and absolute sizes, and, again, there is no way to know which might be better.

Lastly, Ghaidarov considered three variations with different equity values and this same capital structure. In Table 6 we summarize his results and compare them with those produced by the approach presented in this article. For completeness we include the 1st example as well. If there was a single class of security, the DLOM for this example would be 21.9%. We also include the comparative results achieved by adjusting the firm value by this DLOM and then allocating the adjusted value. The right-hand column compares the total value of the security discounts as a percentage of the total value of equity with this value.[11] We believe that the results for these three methods are sufficiently close that it is difficult to argue precedence for one over the other. Appraisers may have personal preferences or may choose the easiest to implement.

These examples have all dealt with allocation of a marketable value of equity among securities and the adjustment of those security values for a lack of marketability. It is often the case that we perform CCA analyses based on the value of a transaction in a non-marketable security and price all other securities in the capital structure relative to that security. In this case, the security-specific marketability adjustment is relatively straightforward. Discussion surrounding the Working Draft of AICPA Accounting and Valuation Guide aid has given rise to what is referred to as the ‘‘differential DLOM’’ or ‘‘incremental DLOM’’ approach. In applying this perspective, the appraiser argues that the observed value of a security, with respect to which other securities are being priced, is a non-marketable value, and, therefore, there is a marketability adjustment implicit in

Table 6
Summary of DLOMs for Four Cases and Three Methods

	Ghaidarov			
Total Firm Value	Series A	Series B	Common	Aggregate
$5,000,000	13.8%	9.8%	50.6%	24.1%
$7,000,000	17.3%	6.6%	39.1%	23.0%
$10,000,000	20.8%	7.8%	30.2%	22.2%
$20,000,000	22.1%	18.6%	23.1%	22.0%
	This article			
$5,000,000	16.4%	8.2%	40.9%	21.5%
$7,000,000	18.2%	7.9%	34.1%	21.7%
$10,000,000	20.2%	10.3%	28.6%	21.8%
$20,000,000	21.9%	18.2%	23.3%	21.9%
	Enterprise adjustment			
$5,000,000	17.3%	11.3%	38.1%	21.9%
$7,000,000	18.4%	10.3%	32.8%	21.9%
$10,000,000	20.1%	11.7%	28.3%	21.9%
$20,000,000	21.8%	18.1%	23.4%	21.9%

Table 7A
Contingent Claims Backsolve for Value of Series B

S	$8,652,177	$8,652,177	$8,652,177	$8,652,177		
X	$0	$4,000,000	$7,000,000	$12,000,000		
R	4.88%	4.88%	4.88%	4.88%		
σ	50.0%	50.0%	50.0%	50.0%		
T	2.000	2.000	2.000	2.000		
d_1	39.363	1.583	0.791	0.029		
C	$8,652,177	$5,224,704	$3,409,675	$1,717,581		
Call Spread	$3,427,473	$1,815,030	$1,692,093	$1,717,581		
N(d_1)	1.000	0.943	0.786	0.512		
Delta Spreads	0.057	0.158	0.274	0.512		
		Ownership claim				
Series A	50%	0%	40%	33%		
Series B	50%	0%	0%	17%		
Common	0%	100%	60%	50%		
		Value of ownership claim			Total	
Series A	$1,713,736	$0	$676,837	$572,527	$2,963,101	
Series B	$1,713,736	$0	$0	$286,264	$2,000,000	
Common	$0	$1,815,030	$1,015,256	$858,791	$3,689,076	
		Security delta			Total	Volatility
Series A	0.028	0.000	0.110	0.171	0.309	45.0%
Series B	0.028	0.000	0.000	0.085	0.114	24.6%
Common	0.000	0.158	0.164	0.256	0.578	67.8%

its value. An example might be the issuance of securities to investors or the purchase of the securities of one private company by another.[12]

[12]We recognize that marketability is a relative, not an absolute, concept. S&P 500 stocks are highly marketable, while other common stocks may be less so, even though they trade on an exchange. Houses are less marketable than most traded common stocks but likely more marketable than apartment buildings. Because the literature on DLOMs uses traded common stocks as its benchmark, I believe it is quite reasonable to refer to privately held equity securities as non-marketable, while recognizing there are different degrees of non-marketability among private securities. If one can quantify those degrees, the analysis described in the next section can accommodate them.

Incremental DLOM

The purpose of this section is to demonstrate that the proposed incremental DLOM is appropriate because it is identical to valuing each security as if it were marketable and then applying its full DLOM. To do this we assume three securities, Series A Convertible Preferred, Series B Convertible Preferred, and Common. Series A and B share the liquidation preference pari passu and split it 50:50. When Series A converts, Common and Series A share in the proportion 40:60. When Series B converts, the proportions for Series A, Series B, and Common are 33.33:16.67:50.[13]

[13]These proportions match the example in Table 5A because we are going to extend it; the argument does not depend on specific values.

Table 7B
Calculation of DLOMs Using Protective Puts

	Protective Puts		
	Series A	Series B	Common
S	$2,963,101	$2,000,000	$3,689,076
X	$2,963,101	$2,000,000	$3,689,076
r	4.88%	4.88%	4.88%
σ	45.0%	24.6%	67.8%
T	2.000	2.000	2.000
Put	$576,275	$180,351	$1,129,809
Full DLOM	19.4%	9.018%	30.6%
Incremental DLOM	11.5%	0.0%	23.7%
Non-marketable Values	$2,623,392	$2,000,000	$2,812,924

Table 8A
Contingent Claims Backsolve for Value of Series B

S	\$9,509,721	\$9,509,721	\$9,509,721	\$9,509,721		
X	\$0	\$4,396,452	\$7,693,791	\$13,189,357		
r	4.88%	4.88%	4.88%	4.88%		
q	0.00%	0.00%	0.00%	0.00%		
σ	50.0%	50.0%	50.0%	50.0%		
T	2.000	2.000	2.000	2.000		
d_1	39.497	1.583	0.791	0.029		
c	\$9,509,721	\$5,742,541	\$3,747,618	\$1,887,816		
Call Spread	\$3,767,180	\$1,994,923	\$1,859,802	\$1,887,816		
N(d_1)	1.000	0.943	0.786	0.512		
Delta Spreads	0.057	0.158	0.274	0.512		
		Ownership claim				
Series A	50%	0%	40%	33%		
Series B	50%	0%	0%	17%		
Common	0%	100%	60%	50%		
		Value of ownership claim			Total	
Series A	\$1,883,590	\$0	\$743,921	\$629,272	\$3,256,783	
Series B	\$1,883,590	\$0	\$0	\$314,636	\$2,198,226	
Common	\$0	\$1,994,923	\$1,115,881	\$943,908	\$4,054,712	
		Security delta			Total	Volatility
Series A	0.028	0.000	0.110	0.171	0.309	45.0%
Series B	0.028	0.000	0.000	0.085	0.114	24.6%
Common	0.000	0.158	0.164	0.256	0.578	67.8%

We use the following variables in the analysis: *E* is the total equity of the firm, and X_0, X_1, X_2 and X_3 are the strike price breakpoints in the CCA; they are, respectively, an arbitrary small positive value close to zero, the total liquidation preference, the value at which Series A converts, and the value at which Series B converts.

The value of a BSM call option is $BSM(E,X_i,r,T,\sigma)$.

Throughout we assume that the risk-free rate of return, time to maturity, and volatility are constant, so we write $BSM(E,X_i,r,T,\sigma) = BSM(E,X_i)$. The value of a BSM call option is scale dependent. Where *q* is a constant,

$$qBSM(E,X_i) = BSM(qE,qX_i).$$

The DLOMs[14] for the three securities are $DLOM_{P_A}$, $DLOM_{P_B}$ and $DLOM_C$.

We define a constant: $y = \frac{1}{1 - DLOM_{P_B}}$.

We assume a non-marketable transaction in Series B at a price P_B. The marketable value of Series B is yP_B

[14]The argument is independent of how the DLOMs are calculated.

Table 8B
Calculation of DLOMs Using Protective Puts

	Protective Puts		
	Series A	Series B	Common
S	\$3,256,783	\$2,198,226	\$4,054,712
X	\$3,256,783	\$2,198,226	\$4,054,712
r	4.88%	4.88%	4.88%
q	0.00%	0.00%	0.00%
σ	45.0%	24.6%	67.8%
T	2.000	2.000	2.000
d_1	0.472	0.455	0.581
d_2	−0.165	0.107	−0.377
Full DLOM	19.4%	9.018%	30.6%
Non-marketable Values	\$2,623,392	\$2,000,000	\$2,812,924

Table 9
DLOMs for a Range of Times to Liquidity Events and Security Volatilities

	Time to Liquidity Event				
	Chaffee Protective Put				
Volatility	1	2	3	4	5
20%	7%	10%	12%	14%	15%
40%	15%	21%	25%	29%	31%
60%	23%	32%	38%	42%	46%
80%	30%	41%	49%	55%	59%
100%	38%	51%	59%	65%	69%
	Finnerty Asian Put				
20%	5%	6%	8%	9%	10%
40%	9%	13%	15%	17%	19%
60%	13%	18%	21%	24%	26%
80%	17%	23%	26%	28%	30%
100%	21%	27%	29%	31%	32%

We implement the CCA to determine the prices of Series A and Common. This is accomplished by solving the following set of three equations for the value of E such that the value of the Series B Preferred matches its transaction value.

$$P_A = 0.50(BSM(E,X_0) - BSM(E,X_1)) + 0.40(BSM(E,X_2) - BSM(E,X_3)) + 0.3333BSM(E,X_3)$$

$$P_B = 0.50(BSM(E,X_0) - BSM(E,X_1)) + 0.1667BSM(E,X_3)$$

$$C = 1.00(BSM(E,X_1) - BSM(E,X_2)) + 0.60(BSM(E,X_2) - BSM(E,X_3)) + 0.50BSM(E,X_3)$$

If we multiply each equation by y we have

$$yP_A = 0.40(BSM(yE,yX_0) - BSM(yE,yX_1)) + 0.50(BSM(yE,yX_2) - BSM(yE,yX_3)) + 0.35BSM(yE,yX_3)$$

$$yP_B = 0.60(BSM(yE,yX_0) - BSM(yE,yX_1)) + 0.30BSM(yE,yX_3)$$

$$yC = 1.00(BSM(yE,yX_1) - BSM(yE,yX_2)) + 0.50(BSM(yE,yX_2) - BSM(yE,yX_3)) + 0.35BSM(yE,yX_3)$$

These three equations illustrate that the marketable equivalents of the total equity and the breakpoints are their contract values scaled up by y. Similarly, the marketable values of the securities are equal to the values determine in the CCA, also scaled up by the constant y. Their non-marketable values are

$$yP_A(1 - DLOM_A) = P_A\frac{(1 - DLOM_A)}{(1 - DLOM_B)} \text{ and}$$

$$yC(1 - DLOM_C) = C\frac{(1 - DLOM_C)}{(1 - DLOM_B)}$$

Therefore, incremental DLOMs for Series A and Common are, as defined in the Working Draft,

$$1 - \frac{(1 - DLOM_A)}{(1 - DLOM_B)} \text{ and } 1 - \frac{(1 - DLOM_C)}{(1 - DLOM_B)}$$

We illustrate this result by modifying the example displayed in Table 5A. The modification is that instead of having an equity value of \$5 million, we assume that the Series B Preferred is just being issued to investors in the private market at its face value \$2 million. As such, the non-marketable value of Series B is \$2 million. We want to calculate the non-marketable values of Series A and the Common. To do that, we perform a backsolve calculation that determines the value of total equity consistent with the transaction and allocates that value among the three securities. Table 7A indicates that the total equity value is \$8.65 million and that the allocations to Series A, Series B, and Common are \$2.96, \$2.00, and \$3.69 million, respectively. Table 7B displays the calculation of the full DLOMs and the incremental DLOMs and the non-marketable values of the three securities.[15]

[15]For clarity note that the incremental DLOM for common is 1 − (1 − .306)/(1 − .090) = 23.7%.

Table 10
DLOM Implied Required Rates of Return for Non-marketable Securities When the Marketable Security Required Rate of Return Is 20%

	Time to Liquidity Event				
	Chaffee Protective Put				
Volatility	1	2	3	4	5
20%	29.6%	26.6%	25.3%	24.5%	23.9%
40%	41.6%	35.1%	32.2%	30.5%	29.4%
60%	55.8%	45.0%	40.4%	37.7%	35.8%
80%	72.5%	56.8%	50.1%	46.2%	43.4%
100%	92.3%	70.6%	61.5%	56.0%	52.1%
			Finnerty Asian Put		
20%	25.8%	24.1%	23.3%	22.9%	22.6%
40%	32.0%	28.4%	26.8%	25.8%	25.2%
60%	38.5%	32.7%	30.1%	28.5%	27.4%
80%	45.1%	36.7%	32.9%	30.5%	28.8%
100%	51.6%	40.1%	34.8%	31.6%	29.5%

Lastly, Table 8A displays the fully marketable values and allocations that are equivalent to the results in Table 7A. Four changes in Table 8 are most important and drive the other changes in values: the total equity value, \$8.65 million, and the three breakpoints, \$4, \$7, and \$12 million, have been divided by the expression one minus the DLOM for Series B Preferred: (1 − .091). For example, the total equity is equal to the value from Table 7A, \$8.65 million, divided by 0.89, which gives total equity in Table 8A a value of \$9.51 million. Table 8B displays the adjustments for lack of marketability. Note that all three securities receive full DLOM adjustments with the result that all have the same non-marketable values as shown in Table 7B, which is, of course, the point in providing Table 8.

Closing Thoughts

Ghaidarov proposes a method for estimating DLOMs using protective puts that is somewhat more complex than existing practice. Its merit is that it provides different DLOMs for different securities in the capital structure. It also eliminates the inconsistency of assuming that both the asset underlying the CCA analysis and the securities that are part of the capital structure have lognormally distributed values. We illustrate another method for estimating differential DLOMs based on individual security volatilities. This method is executed using exactly the same framework as a CCA. Consequently, it is likely easier to implement than Ghaidarov's method, while producing very similar results. We also show that, at least for the capital structures investigated by Ghaidarov, a very similar result is achieved by applying the DLOM to the enterprise value and allocating that reduced value. Lastly, we have shown that when the valuation and allocation process begins with a non-marketable security transaction, the proposed incremental DLOM approach is identical to applying a full DLOM to a marketable value for each security.

This article has emphasized technical analysis. We will close with some non-technical observations about the 800-pound elephant in the middle of the room, which involves asking the question: ''Never mind the arithmetic, what level of DLOM makes sense?'' The importance of this question is highlighted in Table 9, which reports the DLOMs for a range of volatilities and times to liquidity events for Chaffee's protective put model and corresponding values for Finnerty's Asian put model. Both of these models are widely used and accepted, yet they provide dramatically different DLOMs: The protective put DOLMs are approximately 1.5 to 2.2 times as large as the Asian put values. Can we reasonably endorse both?

Another way to express these differences is in terms of their effects on the expected rates of return of the securities. Assume that Table 9 refers to a security that has an expected rate of return of 20% on a marketable basis. If the volatility is 60% and the time to liquidity event is four years, then the DLOM is 42% for the protective put. Such a security, if acquired at a 42% discount, offers an expected rate of return of 38%, as shown in Table 10: $37.7\% = \left(\frac{1.20^5}{1-42\%}\right)^{0.2} - 1$. The question we need to ask is whether it is plausible to believe that the private security holders require an extra

17.7% per year to compensate for the lack of marketability over a four-year period. Table 10 provides the corresponding expected rates of return implied by the two put models for cases reported in Table 9. For a one-year term, the models imply expected rates of return ranging from 25% to 92%. For five-year terms the expected rates of return range from 23% to 52%, with the protective put expected returns much larger than those implied by the Asian put model for higher volatilities.

Current practice with respect to financial reporting places strong emphasis on put models. This is, at least in part, a response to remarks made by the Associate Chief Accountant of the SEC at the 2004 Thirty-Second AICPA National Conference on Current SEC and PCAOB Developments. These remarks emphasized the influence of the duration of restrictions and volatility and objective support for DLOMs. Because put models are objective and incorporate duration and volatility, they appear to fit the SEC requirements. As implemented, however, the subjective choice of which model and which volatility can produce DLOMs that are very large and inconsistent across models. Is this a better result than when we emphasized appraiser judgment and empirical observations? Is it time to ring-fence DLOMs?

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