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Valuing Warrants with Multiple Exercise Prices, and Warrants on Convertible Preferred Stock

by

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Introduction

This paper addresses two warrant valuation topics: 1) warrants with multiple exercise prices and 2) valuing warrants on convertible preferred stock. We address the issue of multiple exercise prices because, while the problem is not common, neither is its solution readily available. This section also allows us to link warrant valuation with contingent claims analysis (CCA). We analyze warrants on convertible preferred to address two outstanding practice questions: Can these warrants be valued using a BSM equation outside the capital structure? If they can be, what volatility is appropriate?

Briefly, we show that 1) there is a simple extension of the modified BSM equation that incorporates multiple warrant issues with different exercise prices and 2) warrants on preferred stock can be reasonably be valued either inside the capital structure or in a simple BSM equation but that the latter approach requires the use of a preferred specific volatility.

Warrants with Multiple Exercise Prices

In addition to providing a method to value warrants with more than one exercise price, this section also links warrant valuation and contingent claims analysis (CCA). We illustrate the fundamental result by considering a firm with two issuances of outstanding warrants with different exercise prices but the same maturity. In a CCA, the value of an enterprise is allocated among securities by partitioning aggregate value into a set of call option spreads. We use the following symbols to describe the process.

S and N are the price and number of shares of the common stock;

M_1 and M_2 are the number of warrants with exercise prices of X_1 and X_2 ($X_1 < X_2$);

W_1 and W_2 are the values of warrants with exercise prices of X_1 and X_2;

C_1 and C_2 are the BSM values of call options on the value of the firm with exercise values equal to NX_1 and $(N + M_1)X_2$; and

V is the value of the firm's common stock and warrants, $V = NS + M_1W_1 + M_2W_2$.

In a CCA analysis, the value of the firm is partitioned into three call option spreads:

$C_0 - C_1$, $C_1 - C_2$, and $C_2 - C_\infty$.

Note that $C_0 = V$ and $C_\infty = 0$. Table 2 displays the percentage of the value of each of these call spreads that is claimed by each security.

[1] Not for quotation without permission.

Table 1
Allocation of Call Spreads in a CCA

	$V - C_1$	$C_1 - C_2$	C_2
NS	N/N	$N/(N + M_1)$	$N/(N + M_1 + M_2)$
M_1W_1		$M_1/(N + M_1)$	$M_1/(N + M_1 + M_2)$
M_2W_2			$M_2/(N + M_1 + M_2)$

We can generalize this result to any number of tranches of warrants by adding more call spreads and increasing the number of warrants in the denominators. Also, note that if $M_2 = 0$, we have a familiar result:

$M_1W_1 = C_1[M_1/(N + M_1)]$,

and the per share warrant value is:

$W_1 = C_1/(N + M_1)$.

If we multiply right-hand side of the above equation by N/N we have:

$W_1 = (C_1/N)N/(N + M_1)$.

The expression (C_1/N) is the per share call option value so this result is identical to that we used in the previous section: The warrant can be valued as $N/(N + M)$ fraction of a call option where the underlying stock price in the BSM formula is the value of common stock plus the value of the number of warrants per share of common stock.

Warrants on Convertible Preferred Stock

This section addresses an issue confronted by appraisers and auditors: How should warrants on preferred stock be valued? To address this question, we compare three alternative approaches to valuing warrants on convertible preferred:

1) The preferred, common and all warrants are valued in a CCA framework.

2) The preferred and common are valued in a CCA and the warrants are valued outside the CCA using the BSM formula with derived volatilities for each preferred stock.

3) The preferred and common are valued in a CCA and the warrants are valued outside the CCA using the BSM formula with all of the volatilities equal to the asset volatility.

The first approach is conceptually most consistent, but also most complex. The second and third approaches are simpler because the warrant allocation occurs outside the CCA. The chief advantage of this change is that the time to a liquidity event and the

warrant time to expiration need not match.[2] In addition to its simplicity, this method allows the capture of the full time value of the warrant. We apply the same constraints to value allocation as in the first approach. For example, if we are calibrating the enterprise value to a transaction, we include both preferred and warrants when applying the constraint. If we are allocating a known enterprise value, we require that the total of all securities including warrants match the enterprise value. The volatilities used in the second approach are estimated in the CCA, while in the third approach the asset volatility is applied to all securities.

We examine the results of these three approaches for an example of a firm with common stock and three classes of convertible preferred stock, each with warrants. Each class of the preferred stock has an 8% coupon and equal seniority with respect to their liquidation preferences. We treat dividends as being accrued. There are 7 breakpoints in the allocation of enterprise value: 1) Preferred stock liquidation preferences are met; 2) Preferred A converts; 3) Warrants on Preferred A are exercised; 4) Preferred B converts; 5) Warrants on Preferred B are exercised; 6) Preferred C converts; 7) Warrants on Preferred C are exercised. The volatility of the assets is 40.0% and the estimated time to a liquidity event is 5 years. The Table 2 provides the capital structure details.

Table 2
Capital Structure

	Count	Face	Exercise Price	Coupon
Preferred A	200	$200	$1.00	8.0%
A Warrants	200		$1.50	
Preferred B	300	$600	$2.00	8.0%
B Warrants	300		$3.00	
Preferred C	500	$1,250	$2.50	8.0%
C Warrants	500		$3.75	
Common	1,000			

Table 3 summarizes the valuation results. Panel A in Table 3 assumes a recent transaction in Preferred C and its warrants at $2.50 for a unit of one preferred share and one warrant. Each of the three methods calibrates the enterprise value to that transaction. The value of all securities[3] for the first method is $3,000. Clearly the three methods produce different values for the different securities. In Panel A, the first method produced a higher total value for the securities so that, on average, the other two methods undervalue the securities. In the first method, the volatilities of Series A, B and C preferred stock were 36.7%, 26.8% and 22.7% respectively.[4] These values vary inversely with their exercise prices because the lower the exercise price, the higher the

[2] Not that the CCA implicitly assumes the maturity of all securities matches the liquidity event date. In our examples we explicitly assume the maturity dates of the warrants match the liquidity event date.

[3]When we value the warrants outside the CCA, the "Enterprise" value is the sum of the values of the preferred and the common but does not include the values of the warrants.

[4] These values are determined by the equation $\sigma_S = \mathrm{N}(d_1)(V/S)\,\sigma_V$, with the additional understanding that the delta is a weighted average of the deltas of the call spreads that determine the value of the security.

percentage of a preferred stock's value that it derives from conversion to equity. The second method, which uses a constant 40.0% volatility for the valuation of each of the warrants, overestimates the value of the Series B and C Warrants, even while undervaluing the securities as a group. While the results for methods two and three were directional similar, method three produces values much close to the values calculated in method one.

Panels B and C of Table 3 consider scenarios in which the values of all securities are $4,500 and $1,500 respectively. In each panel, we report volatilities for each security. We calculate theses volatilities using the expression that relates the volatility of the security to the volatility of the underlying asset that we saw earlier:

$\sigma_S = \mathrm{N}(d_1)(E/S)\sigma_E$

The difference here is that the value of the security is a weighted average of a set of call spreads and so the delta shown as $\mathrm{N}(d_1)$, is a weighted average of the deltas of those same call spreads.

In panels B and C, the total values of the securities are the same for all three methods. Consequently each of the three methods must overestimate the values of some securities and underestimate the values of others, relative to the other two methods. These results support two important observations. As was the case for Panel A, the third method more closely matches the results of the first method. When the value of the securities is low relative to the total liquidation preference, the differences are less noticeable because a higher percentage of total value is allocated to the liquidation preferences, and the methods differ less with respect to this allocation.

Summary

We showed that warrants with multiple exercise prices when valued in a CCA framework rely on formulas that are structurally similar to the formula used to value a warrant and adjust for dilution.

The best approach to valuing warrants on preferred stock is to include the warrants in a full CCA. If the warrants are valued outside the capital structure in a BSM equation, it is important to effectively estimate the volatility of the preferred stock and not to revert to the volatility of the assets.

Table 3A
Valuation Results for an Assumed Transaction in Series C Preferred: Enterprise Value is $3,000

	Warrants Inside the CCA			Warrants Outside the CCA Structure					
	Total Value	Per Share Value	Volatility	Total Value	Per Share Value	Volatility	Total Value	Per Share Value	Volatility
"Enterprise"	$2,999.42			$2,301.80			$2,704.86		
Preferred A	$256.98	$1.28	36.7%	$211.14	$1.06	40.0%	$247.86	$1.24	34.6%
Warrants on Pref. A	$73.46	$0.37	81.3%	$56.19	$0.28		$66.85	$0.33	
Preferred B	$617.94	$2.06	26.8%	$531.78	$1.77	40.0%	$600.37	$2.00	26.2%
Warrants on Pref. B	$50.77	$0.17	97.0%	$114.87	$0.38		$76.53	$0.26	
Preferred C	$1,196.06	$2.39	22.7%	$1,041.98	$2.08	40.0%	$1,157.33	$2.31	22.9%
Warrants on Pref. C	$53.95	$0.11	106.4%	$208.02	$0.42		$92.61	$0.19	
Common	$750.27	$0.75	66.9%	$516.91	$0.52	40.0%	$699.30	$0.70	62.9%
Securities Total	$2,999.42			$2,680.89			$2,694.54		

Table 3B
Valuation Results for an Assumed Enterprise Value of $4,500

	Warrants Inside the CCA			Warrants Outside the CCA Structure					
	Total Value	Per Share Value	Volatility	Total Value	Per Share Value	Volatility	Total Value	Per Share Value	Volatility
"Enterprise"	$4,500.00			$3,707.51			$3,996.95		
Preferred A	$373.28	$1.87	36.9%	$341.12	$1.71	40.0%	$368.52	$1.84	33.5%
Warrants on Pref A	$157.46	$0.79	69.4%	$143.78	$0.72		$149.15	$0.75	
Preferred B	$807.31	$2.69	26.2%	$759.60	$2.53	40.0%	$803.72	$2.68	24.3%
Warrants on Pref B	$126.97	$0.42	84.0%	$245.12	$0.82		$168.30	$0.56	
Preferred C	$1,489.59	$2.98	21.0%	$1,409.84	$2.82	40.0%	$1,477.03	$2.95	20.1%
Warrants on Pref C	$147.87	$0.30	92.7%	$403.59	$0.81		$185.60	$0.37	
Common	$1,397.52	$1.40	56.2%	$1,196.95	$1.20	40.0%	$1,347.69	$1.35	51.6%
Securities Total	$4,500.00			$4,500.00			$4,500.00		

Table 2C
Valuation Results for an Assumed Enterprise Value of $1,500

	Warrants Inside the CCA			Warrants Outside the CCA Structure					
	Total Value	Per Share Value	Volatility	Total Value	Per Share Value	Volatility	Total Value	Per Share Value	Volatility
"Enterprise"	$1,500.00			$1,358.93			$1,425.82		
Preferred A	$136.15	$0.68	36.8%	$126.56	$0.63	40.0%	$132.54	$0.66	36.2%
Warrants on Pref A	$14.67	$0.07	105.4%	$17.08	$0.09		$15.01	$0.08	
Preferred B	$375.55	$1.25	31.2%	$351.37	$1.17	40.0%	$365.47	$1.22	31.2%
Warrants on Pref B	$7.64	$0.03	122.3%	$41.97	$0.14		$22.68	$0.08	
Preferred C	$764.53	$1.53	29.4%	$714.38	$1.43	40.0%	$741.26	$1.48	29.7%
Warrants on Pref C	$6.84	$0.01	132.5%	$82.03	$0.16		$36.49	$0.07	
Common	$194.63	$0.19	89.7%	$166.61	$0.17	40.0%	$186.55	$0.19	85.1%
Securities Total	$1,500.00			$1,500.00			$1,500.00		