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Comparing Three Convertible Debt Valuation Models”[1]

by

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Comparing Three Convertible Debt Valuation Models

In this article we 1) describe and illustrate the implementation of three convertible debt valuation models, 2) show how their values for convertible debt respond to changes in the underlying valuation parameters, 3) examine the effects of changing each of the models such that the credit spread and the probability of default are not constant but vary inversely with the stock price and 4) measure and compare the accuracy of each model when it is calibrated to convertible debt issuance prices and then used to forecast the convertible debt price one year later.

Introduction[2]

Appraisers frequently value non-traded convertible debt, either to estimate the fair value of the debt component of the convertible or to estimate the value of one or more derivatives embedded in the convertible debt. There is an extensive literature on the valuation of convertible debt and a large number of models have been proposed. In our experience, appraisers most frequently use a credit spread model developed by Tsiveriotis and Fernandes when they were at Morgan Stanley (the "TF" model). Less frequently appraisers use a probability-weighted discount rate model developed by Bardhan, Bergier, Derman, Dosembet, and Kani when they were at Goldman Sachs & Company (the "GS" model). These two models were developed independently in the same time period as practical approaches to the valuation of a complex hybrid security. The TF model figures much more prominently in the literature having been cited 295 times as compared to 28 times for the GS model. More recently, appraisers have begun to use "jump-to-default" models. A paper by Milanov and Kounchev (the "MK" model) describes one good example of such a model.

The TF model values the conversion feature in an option-pricing risk-neutral framework and values the principal and interest payments in a "real world" discounted cash flow framework. Consequently, the model discounts conversion values at the risk-free rate and principal and interest payments at a credit-adjusted rate. The GS model differs in that it discounts all cash flows by a weighted average of the risk-free rate and the credit adjusted rate with the weights determined by the probabilities of conversion to common stock and redemption. Both models implement the calculation of cash flows and their discounting with sets of lattices that we describe below. The foundation for these lattices is a common stock price lattice.[3] In contrast, the MK model values all payments in a risk-neutral framework. The foundation of this model is a CRR stock price lattice that adds a jump to default at each date. Necessarily, this model is augmented with assumptions about recovery rates in the event of default.

The purpose of this article is to demonstrate the implementation of the models, Illustrate how they respond to changes in inputs and compare their performance. Our perspective on performance is that of an appraiser who is pricing nontraded convertible debt by calibrating a model to the issuance price and is concerned with the performance of the developed model at a later date. This contrasts with previous empirical studies that examine the performance of models in pricing traded convertible debt based on inputs derived independently of the convertible debt trading price.[4]

We first describe the mechanics of each model and illustrate each with a numerical example. Second, we compare how the values of the three models vary with changes in the valuation parameters. Third, we examine the effects of changing each of the models such that the credit spread and the probability of default are not constant but vary inversely with the stock price. Put another way, we introduce a credit spread lattice and a probability of default lattice instead of using constant values for all future stock

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[2] Tsiveriotis K. and C. Fernandes. Valuing Convertible Bonds with Credit Risk. Journal of Fixed Income, September 1998, Vol. 8, pp. 95-102. Bardhan, I., A. Bergier, E, Derman, C. Dosembet, and I Kani. "Valuing Convertible Bonds as Derivatives." Goldman Sachs, *Quantitative Strategies Research Notes*, (1994). Milanov, K. and O. Kounchev. Binomial Tree Model for Convertible Bond Pricing within Equity to Credit Risk Framework, working paper Institute of Mathematics and Informatics, Bulgarian Academy of Science, June 2012. Hull presents a similar model. See also Spiegeleer and Schoutens.

[3] The TF model uses a Cox, Ross, Rubinstein lattice, while the GS model uses a Jarrow and Rudd model. To facilitate comparisons, we employ the CRR lattice for the GS model. Cox, J. C., S. A. Ross, and M. E. Rubinstein. "Option Pricing: A Simplified Approach." Journal of Financial Economics, 3 (September 1979), 229–263 and Jarrow, R. A. and A. Rudd. Option Pricing. Homewood, IL: Irwin, 1983.

[4] See Zabolotnyuk, Y, R. Jones, and C. Veld. "An Empirical Comparison of Convertible Bond Valuation Models." Financial Management, Vol. 39, pp. 675 – 706 for one such article and a review of others.

prices. While this is conceptually an appealing improvement for each model, our illustrations suggest that the improvement in the model is likely not worth the cost of the added complexity. Fourth, we use market data to provide further insight into the valuation results that each model provides. We find that the three models perform almost identically as price forecasting models. In addition, our results suggest that price forecasting is better when one continues to use the original calibrated parameters for the volatility of the underlying equity and credit risk rather than holding credit risk constant and updating the volatility.

Mechanics of the TF Model

The foundation of this model is a binomial stock price lattice. A second lattice tracks the value of the convertible debt based on its conversion value, along the paths where it is converted. A third lattice tracks the value of the convertible debt based on its interest payments and the payment of principal at maturity along the price paths where it is redeemed. A fourth lattice tracks the combined conversion and debt values. We illustrate this model with an example: The face value of the convertible debt is $100. It has a 5-year term and a coupon rate of 3.28%, paid annually. The risk-free rate and the credit spread are 2% and 8% respectively.[5] The debt converts into 1 share of common stock with a current price of $80. The annual volatility of the common stock is 30%. For this illustration, we model 1-year time steps in the lattices. See Table 1.

We use a CRR stock price lattice. At each node in the lattice the stock price can move up by the factor u or down by the factor d. The probabilities of the up and down moves are p_u and p_d :

$$u = e^{\sigma\sqrt{\Delta t}}; \quad d = \frac{1}{u}, \quad p_u = \frac{e^{r\Delta t} - d}{u - d}, \; p_d = 1 - p_u ,$$

r is the risk-free rate and Δt is the time between nodes. For example, Table 1 illustrates that in the first year the stock price either moves from 80.00 up to 107.99, with probability, p_u = 46% or down to 59.27 with a probability p_d = 54%.

The conversion lattice is connected to the stock price lattice at the last date with the values being the conversion value when it exceeds face value and zero otherwise. The earlier nodes are the discounted present values in a backward recursive manner. In the Table 1 we have excluded consideration of more complex features such as forced early conversion.

The straight debt lattice values are also calculated in a backward recursive manner with the values at the last date being face value plus interest when the face value exceeds the conversion value and interest otherwise. The earlier nodes are the discounted present values calculated in a backward recursive manner, with the discount rate being the risk-free rate plus a credit spread.

The total value lattice is, in this relatively simple example, the sum of the conversion and the straight debt lattices. In more complex examples it is used to track the effects on value of features such as forced conversion.

Table 1 produces a value for the convertible that is 3.44% above par. When we expand the model to 60 steps the value is almost exactly par, which was the objective in choosing the inputs.

[5] All interest rates are compounded semi-annually.

Table 1
The Four Lattices in the TF Model

Date	0	1	2	3	4	5	0	1	2	3	4	5
	Stock price lattice						Conversion value lattice					
	80.00	107.99	145.77	196.77	265.61	358.54	55.54	89.47	136.91	196.77	265.61	358.54
		59.27	80.00	107.99	145.77	196.77		28.86	52.62	91.29	145.77	196.77
			43.90	59.27	80.00	107.99			9.81	21.82	48.54	107.99
				32.53	43.90	59.27				0.00	0.00	0.00
					24.10	32.53					0.00	0.00
						17.85						0.00
	Straight debt value lattice						Total value lattice					
	47.80	39.14	23.25	8.95	6.26	3.28	103.34	128.62	160.15	205.72	271.86	361.82
		64.18	53.34	33.07	6.26	3.28		93.04	105.96	124.36	152.02	200.05
			78.83	73.92	55.37	3.28			88.64	95.75	103.91	111.27
				91.22	96.96	103.28				91.22	96.96	103.28
					96.96	103.28					96.96	103.28
						103.28						103.28

Mechanics of the GS Model

The foundation of this model is the same binomial price lattice. A second lattice tracks the probability of conversion at each node. A third lattice calculates a node specific discount rate based on the probability of future conversion. A fourth lattice tracks the convertible bond value. We illustrate this model in Table 2 using the same parameters used to illustrate the TF model.

The probability lattice shows entries at date 5 are either 100% or 0%, depending on the terminal stock price because at that date we know whether conversion occurs. Corresponding to these results, the discount rate lattice shows the risk-free rate, 2%, for those cash flows where conversion occurs and the credit-adjusted rate, 10%, for those where it does not occur. We illustrate the valuation of the convertible bond with these examples:

$$152.26 = p_u 200.50(1.01)^{-2} + p_d 111.27(1.01)^{-2}$$

$$104.02 = p_u 111.27(1.01)^{-2} + p_d 103.28(1.05)^{-2}$$

$$96.96 = p_u 103.28(1.05)^{-2} + p_d 103.28(1.05)^{-2}, \text{ and}$$

$$95.71 = p_u 104.02(1+.5(6.33\%))^{-2} + p_d 96.96(1.05)^{-2}, \text{ where}$$

$$6.33\% = (46\%)(2\%) + (1-46\%)(10\%)$$

Note that the discount rate, 6.33%, is a weighted average of the risk-free rate and the risky rate with the weights being taken from the Probability of conversion lattice. At this particular valuation node the probability of conversion at the next time period is 46%, which is the probability of the stock price moving up from 80 to 107.99, at which point conversion would occur. Likewise the probability of conversion of 71% at time period 3 is equal to 46%(100%) + (1 – 46%)(1 – 46%).

The convertible bond value in this example is 106.06 and when we expand the model to 60 steps it is 97.59, 2.41% lower than that calculated in the TF model. [6]

[6] Lattices with this few steps do not provide precise estimates of value. Also, our prior expectation was that with the same parameters the TF and GS models would produce a similar but not identical convertible debt value. For these parameters they differ by 2.41%.

Table 2
The Four Lattices in the GS Model

Stock price lattice

Date	0	1	2	3	4	5
	80.00	107.99	145.77	196.77	265.61	358.54
		59.27	80.00	107.99	145.77	196.77
			43.90	59.27	80.00	107.99
				32.53	43.90	59.27
					24.10	32.53
						17.85

Weighted-average discount rate lattice

Date	0	1	2	3	4	5
		3.87%	2.20%	2.00%	2.00%	2.00%
		9.09%	6.62%	3.27%	2.00%	2.00%
			9.93%	9.23%	6.33%	2.00%
				10.00%	10.00%	10.00%
					10.00%	10.00%
						10.00%

Probability of conversion lattice

Date	0	1	2	3	4	5
	42%	62%	84%	100%	100%	100%
		25%	44%	71%	100%	100%
			10%	21%	46%	100%
				0%	0%	0%
					0%	0%
						0%

Convertible debt value lattice

Date	0	1	2	3	4	5
	106.06	129.45	161.40	206.42	272.10	361.82
		92.15	105.96	124.64	152.26	200.05
			88.18	95.71	104.02	111.27
				91.22	96.96	103.28
					96.96	103.28
						103.28

Mechanics of the MK Model

This model incorporates debt risk by modeling default in a risk-neutral framework. At each date the stock price can move up or down, just as in Table 1, but it can also move to a default value that is modeled as a percentage of the value in the node from which it is moving. Therefore, at each node in the lattice the stock price can move up by the factor *u*, down by the factor *d*, or down by the factor *b*. The probabilities of the moves are, p_u , p_d , and p_b. In the MK model default is a Poisson process with the probability of default over any time interval being $1-e^{-\lambda\Delta t}$. This introduces three new inputs, the risk-neutral default intensity, λ, the recovery rate on the debt in the event of default and the response of the stock price to default, *b*. The default intensity and recovery rate relate to the credit spread in the TF model and the size of the stock price decline in the event of default is a new parameter. In the MK risk-neutral framework, the mathematical expressions for the variables *u* and *d* are the same as the TF model but the probabilities are different.

$$u = e^{\sigma\sqrt{\Delta t}}; \quad d = \frac{1}{u}, \quad p_u = \frac{e^{r\Delta t} - de^{-\lambda\Delta t} - b\left(1-e^{-\lambda\Delta t}\right)}{u-d}, \quad p_d = 1 - p_u - p_b, \quad p_b = 1 - e^{-\lambda\Delta t}.$$

The stock price lattice in Table 3 is built with the same inputs as those used for Table 1 and the addition of a default intensity of 14.6%, a recovery rate of 40% and a jump to default of -70% meaning *b* = 0.30. To make Tables 1 and 2 comparable, the default intensity is based on the risk-free rate and the credit spread used in the TF model, namely it is the risk-neutral default intensity consistent with a 5-year, 3.28% coupon debt with a 40% recovery rate that is priced to yield 10% (2% + 8%). In Table 3, the stock price moves from 80.00 either up to 107.99 or down to 59.27 or into bankruptcy with a value of 24.00 =o.30(80).[7] Likewise, at date 1 if the stock price moved down to 59.27 it can next move up to 80.00, down to 43.90 or into bankruptcy at 17.78 = 0.30(59.27). In the bankruptcy states the value of the convertible debt is the higher of its conversion value or the recovery value which is 40.

Because all of the discounting in this model is at the risk-free rate, the model requires only one other lattice, one that tracks the value of the convertible debt. At date 5 the value of the convertible debt is the higher of its value in conversion or its face value, plus interest. At each of the other nodes, the value of the convertible is calculated in a backward recursive manner as the discounted probability-weighted values it can assume at the next date. In this illustration, the value of the convertible debt is 101.30 and

[7] This feature tends to offset the undesirable effect of the probability of default being unchanged as the stock price increases.

for the 60-step version of the model the value is 102.94, 2.94% higher than the value produced by the TF model.

Table 3
The Two Lattices in the MK Model

Date	0	1	2	3	4	5	0	1	2	3	4	5
	Stock price lattice						Convertible debt value lattice					
	80.00	107.99	145.77	196.77	265.61	358.54	101.30	125.74	157.79	205.18	271.67	361.82
		59.27	80.00	107.99	145.77	196.77		90.73	102.90	121.75	151.83	200.05
			43.90	59.27	80.00	107.99			86.16	92.39	100.46	111.27
				32.53	43.90	59.27				90.01	96.10	103.28
					24.10	32.53					96.10	103.28
						17.85						103.28
		24.00	32.40	43.73	59.03	79.68						
			17.78	24.00	32.40	43.73						
				13.17	17.78	24.00						
					9.76	13.17						
						7.23						

Comparison of the Changes in Values of the Three Models as Parameters Change

Having illustrated the mechanics of the models, we now compare the changes in their values when we vary their parameters. To increase the precision of the results we discuss, they are based on 60-step implementations of each model. We adopt the parameters discussed in the previous section as a base case. We measure the effect on the price of the convertible for two additional credit spreads, 4% and 12%, for levels of price decline -70% and -100% (b = 0.30 and b = 0.0) and for two tenors, 5 years and 10 years. For the 10-year tenor, the coupon rate is decreased to 2.93% so that the TF model calibrates to par. Table 4 presents the results.

We discuss the results for the TF and GS models first. For both the 5-year term and the 10-year term, the base case value of the convertible debt is par for TS by design and close to par for the other two models. For each term we increase and decrease the credit spread by 400 basis points. As we would expect, the value of the convertible increases when the credit spread declines and decreases when the credit spread increases. For these parameters the changes in values are 10.8% and -8.6% for the 5-year term and 15.0% and -10.4% for the 10-year term. For the GS models the corresponding changes in value are similar, but somewhat larger in both directions, 11.6% and -10.0% for the 5-year term and 17.8% and -14.0%.

The MK model values the convertible at very close to par for the base case when the equity jump is -70%, 102.94 and 100.60 for the 5-year and 10-year terms. However, if we select an equity jump of -100%, the value of the convertible is substantially larger, 114.59 and 123.85. It is important to understand this counter intuitive result. The total volatility of the common stock price is a function of the familiar volatility that we have used in the TF model, $\sigma = 30\%$ in our example, and the size of the equity jump and the probability of default. Specifically, total stock price volatility for the MK model[8] is $\sqrt{\sigma^2 + \lambda b^2}$. When b is 0.30, total volatility is 42.5% and when b is 0.00, total volatility is 55.5%. This increase in volatility explains why the value of the convertible debt increases when the equity price jump is -100% rather than -70%. This point also highlights that the two models are not comparable with respect to volatility. For the MK model to have a total volatility in the base case with a stock price jump of -70%, the usual volatility term, what we have referred to as σ, would have to be 13.6%. For those parameters the value of the convertible would decrease to from 102.94 to 96.48. We will return to this subject when we discuss the estimation of the parameters for the models and the models' relative merits.

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[8] See Milanov and Kounchev who derive this relationship based on the assumption that default and stock price are uncorrelated.

When we decrease the credit spread by 400 basis points, the corresponding default intensity decreases from 14.60% to 8.36%; when we increase the credit spread by 400 basis points the default intensity increases from 14.60% to 57.28%. The effect on the value of the convertible is not as straightforward as for the TF and GS models and depends on the size of the equity price jump because of its effect on volatility as just discussed. We highlight one case: When the equity jump is -100%, a decrease and an increase in the credit spread of 400 basis points increases the value of the convertible debt by almost equal amounts, 1.67 and 1.98. The increase in the value of the convertible in the MK model in response to an increase in the credit spread is a clear and important difference between the MK model and the TF and GS models and we view it as a negative feature of the MK model.

We also compare the values produced by the models for different degrees of moneyness. Table 4 reports the values of the convertible debt for the 5-year and 10-year terms, the two equity jumps, -70% and -100% and three levels of moneyness, 60%, 80% (the base case) and 100%. The changes in values attributable to the change in moneyness (common stock price) are both reasonable and consistent among the three models.

Table 4
Convertible Debt Values for the TF and MK Models for Combinations of Parameters

S/X = 80/100	80%				80%			
Term	5.00				10.00			
Coupon rate	3.28%				2.93%			
Model	TF	GS	MK		TF	GS	MK	
Recovery rate			40.00%				40.00%	
Credit spread/Default intensity	4.00%	4.00%	8.36%		4.00%	4.00%	7.27%	
Equity Jump			-70%	-100%			-70%	-100%
Volatility	30.0%	30.0%	35.1%	43.7%	30.0%	30.0%	35.4%	44.5%
Pure bond	88.40	88.40	88.40	88.40	77.15	77.15	77.15	77.15
Conversion value	22.18	21.26	22.93	27.86	37.84	35.60	36.75	47.22
Convertible	**110.58**	**109.66**	**111.33**	**116.26**	**114.98**	**112.75**	**113.90**	**124.37**
Credit spread/Default intensity	8.00%	8.00%	14.60%		8.00%	8.00%	18.57%	
Equity Jump			-70%	-100%			-70%	-100%
Volatility	30.0%	30.0%	40.2%	55.5%	30.0%	30.0%	42.5%	60.6%
Pure bond	74.05	74.05	74.05	74.05	55.93	55.93	55.93	55.93
Conversion value	25.96	23.63	28.89	40.54	44.07	38.12	44.67	67.91
Convertible	**100.00**	**97.68**	**102.94**	**114.59**	**100.00**	**94.05**	**100.60**	**123.85**
Credit spread/Default intensity	12.00%	12.00%	57.28%		12.00%	12.00%	93.56%	
Equity Jump			-70%	-100%			-70%	-100%
Volatility	30.0%	30.0%	45.7%	67.2%	30.0%	30.0%	74.1%	121.8%
Pure bond	62.35	62.35	62.35	62.35	41.35	41.35	41.35	41.35
Conversion value	29.02	25.07	35.08	54.22	48.24	38.17	47.17	80.56
Convertible	**91.37**	**87.42**	**97.44**	**116.57**	**89.59**	**79.52**	**88.52**	**121.91**

Table 5
Convertible Debt Values for the TF and MK Models for Degrees of Moneyness

Term	5.00				10.00			
Coupon rate	3.28%				2.93%			
Model	TF	GS	MK		TF	GS	MK	
Recovery rate			40.00%				40.00%	
Credit spread/Default intensity	8.00%	8.00%	14.60%		8.00%	8.00%	18.57%	
Equity Jump			-70%	-100%			-70%	-100%
Volatility	30.0%	30.0%	40.2%	55.5%	30.0%	30.0%	42.5%	60.6%
S/X	Convertible Value				Convertible Value			
60/100 = 60%	87.99	86.79	89.54	97.53	80.29	75.32	85.55	104.03
80/100 = 80%	100.01	97.68	102.94	114.59	100.00	94.05	101.04	123.77
100/100 = 100%	115.10	111.83	118.45	132.96	114.23	104.93	117.83	143.64
For S/X = 60%: % change from 80%	-12.0%	-11.1%	-13.0%	-14.9%	-19.7%	-19.9%	-15.3%	-15.9%
For *S* / *X* =100%: % change from 80%	15.1%	14.5%	15.1%	16.0%	14.2%	11.6%	16.6%	16.1%

Addressing a Common Shortcoming of the Models

All three of these models implicitly assume that the probability of bankruptcy is independent of the price of the underlying common stock price. In general, that is an unreasonable assumption. We would expect the credit spread in the TF and the GS models and the default intensity in the MK model to be inversely related to the price of the common stock. In this section of the paper, we examine the effects on value of introducing this relationship.

The results for the TF and GS models are similar and so we discuss the TF model in detail. We express the credit spread as a modified power function with the degree of moneyness as the independent variable. This produces a credit spread lattice in which the credit spread at each node is determined by the power function and the stock price at the corresponding node in the stock price lattice. Credit adjustment of debt cash flows is node (stock price) specific. We identify parameters of the modified power function such that the credit spread for a stock price of 80 is approximately the 8% used in our base case. Figure 1 displays the function.

For the MK model we follow a similar process. In this case, we develop a default intensity lattice that is a function of the stock price. This gives rise to corresponding probability lattices for the each of the three possible price moves, *u*, *d* and *b* and the default intensity and the probabilities are all node (stock price) specific. We identify a function such that the default intensity for a price of 80 is approximately the 14.6% used in the base case. Figure 2 displays the function.[9]

The operative question is whether these changes in the models, which are complex and require judgment with respect to the functional form of the credit spread and default intensity equations, are worth the effort. The results in Table 6 suggest that they are not. For the 5-year convertible debt the price differences caused by the more complex models range from less than 1% to a maximum of 4%. To illustrate this result consider the base case. With a constant credit spread or default intensity the values of the 5-year debt are 100.00 102.94 and 114.59 for the TF model and the two versions of the MK model, respectively. When we introduce the price varying spread and default intensity the values are 98.02, 103.79 and 111.62, differences of only 2.0%, -0.8% and 2.6%.

[9] We selected the functional forms in Figures 1 and 2 numerically such that the price effect of the credit adjustment went to zero as the convertible debt was very deep in the money and to 100% when it was deep out of the money.

Table 6
Convertible Debt Values for the TF and MK Models for Degrees of Moneyness

S/X = 80/100	80%		
Term	5.00		
Coupon rate	3.28%		
Model	TF	MK	
Credit spread/Default intensity	4.00%	8.36%	
Equity Jump		-70%	-100%
Constant	110.58	111.33	116.26
Inverse function of stock price	110.21	112.14	114.92
Credit spread/Default intensity	8.00%	14.60%	
Equity Jump		-70%	-100%
Constant	100.00	102.94	114.59
Inverse function of stock price	98.02	103.79	111.62
Credit spread/Default intensity	12.00%	57.28%	
Equity Jump		-70%	-100%
Constant	91.37	97.44	116.57
Inverse function of stock price	88.49	98.94	112.05

Figure 1
Credit Spread as an Inverse Function of Moneyness

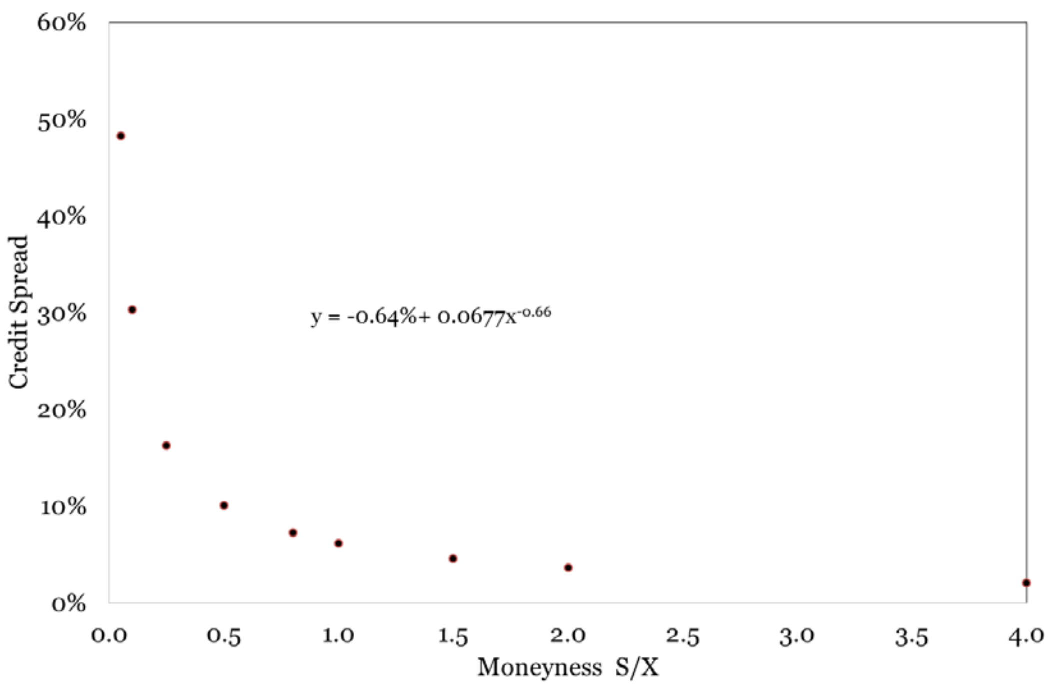
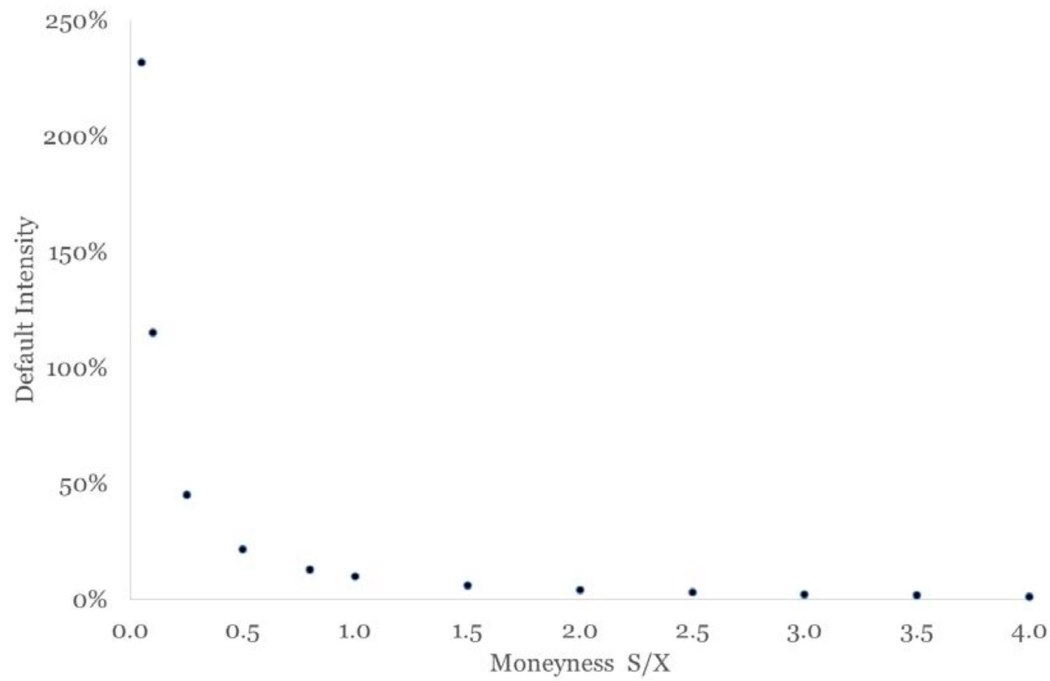


Figure 2
Default Intensity as a Function of Moneyness



Examining the Pricing Performance of the Three Models.

We collected data on convertible debt issued by companies with publicly traded common stock from 2012 to early 2015. After eliminating all callable convertibles, to simplify the valuation, 35 issues remained. We calibrated all three models to market prices at the time of issuance of the debt.[10] The calibration process involved estimating a volatility of the underlying common stock and deriving the credit spreads (TF and GS models) or default intensity (MK model) that produce the price of the convertible debt at issuance. We used the calibrated models to forecast the price of the convertible debt one year after issuance, based on changes in the stock price, interest rate, time to maturity and, possibly, the volatility of the common stock. We compared those forecasted values with the reported market prices of the convertible debt at that time.

We designate F_i and A_i as the forecast and actual percentage changes in value of each convertible debt over the one-year period following its issuance. We measure the forecasting performance in two ways. First, we calculate the average of the forecast errors, $(A_i - F_i)$ and test it for bias and measure its standard deviation as an indication of accuracy. Second, we assess accuracy by estimating the following linear regression:

$$A_i = \hat{a} + \hat{b}F_i + e_i$$

If our forecasts were perfect, we would find $\hat{a} = 0.0$, $\hat{b} = 1.0$ and the R^2 would be 100%.

We summarize the forecasting results in Table 7a for three different approaches to measuring volatility. Our first approach was to measure the historic volatility of each security at the issuance date, calibrate the models, then re-measure volatility at the forecast date, holding the credit spread or default intensity constant between issuance and the forecast. This approach produced negatively biased forecasts. The mean values of $(A - F)$ were –4.4%, –3.9% and –3.6% for the TF, GS and MK models, respectively. All three values were statistically significantly different from 0.0. We also found that the errors were negatively correlated with the initial measurement of volatility.[11]

[10] We used the original models, where the credit risk did not varied with the stock price. To allow for the market to establish reliable values we calibrated the models one week after issuance and, in a few cases where the market prices of the convertible debt were not yet available, either two or three weeks after issuance.

[11] We have not been able to identify an explanation for this result.

This led to our second approach which was to use both the initial volatility and the initial credit spread/default intensity at the forecast date, i. e., to not update the volatility estimate. The mean values of $(A - F)$ for this approach were 0.6%, 0.3% and 0.0% for the TF, GS and MK models, respectively and none was statistically significantly different from 0.0. The standard deviations of $(A - F)$ for the second approach were also smaller than those for the first approach indicating that it was not only unbiased but also more accurate.

The results for the case with constant volatility suggest that consistency of the two inputs may be more important than the precision of their individual estimations. To investigate that we tested a third approach to volatility, namely assuming a single constant volatility of 30% for all securities at both dates. This approach also produced unbiased forecasts with mean values of $(A - F)$ of 0.3%, 0.3% and −0.2% for the TF, GS and MK models, respectively. The accuracy of this third approach, as indicated by the standard deviations of the forecast error is essentially identical to the accuracy of the second approach. This supports the idea that consistency of the two inputs is more important than the precision of their estimations.

When we measure forecast performance in terms of a regression of actual percentage change on forecast percentage change we find the results are very similar for the three methods of measuring volatility and for the three models. Using two different volatilities produces slightly lower R^2 values for the TF and GS models and given that its forecasts are biased, we would tend to reject the idea of trying to update the volatility when holding the measure of credit risk constant. The TF and GS models have slightly higher R^2 values than the MK model but the differences are so small all three models perform essentially identically. This conclusion is further reinforced by the fact that the pair-wise R^2 values between the three forecasts using a single volatility are greater than 99.5%.

In Table 7b we provide summary statistics and individual results for the 35 convertible debt securities in our sample for the analysis using a single volatility. The average tenor of the convertibles at issuance was 8.15 years, the average coupon was 1.56%, the average degree of moneyness was 73% and the average volatility was 43%.[12] The average credit spreads, were 5.85% for the TF model and 4.61% for the GS model. It appears that the credit spread for the TF model is consistently larger than that for the GS model. This means that in the context of partitioning the debt value for financial reporting purposes the TF model will consistently identify a smaller debt value and higher yield than the GS model. The average default intensity was 11.42%. In general, this value translates to an effective yield that is higher and bond value that is lower than either of the credit spread models.

[12] For two of the securities the historical volatility implied negative credit spreads and default intensity and in those two cases we adjusted the volatilities until the credit spreads and default intensity were 0.0%.

Table 7a
Results of Price Forecasting Models

TF

Alternative Volatility Choices	$A - F$ Mean	$A - F$ Standard Deviation	Forecast Regression $\hat{b}$	Forecast Regression R^2
Two different volatilities	-4.39%	8.02%	0.91	90%
One volatility	0.65%	6.93%	0.97	93%
Volatility of 30%	0.30%	6.41%	0.88	93%

GS

	$A - F$ Mean	$A - F$ Standard Deviation	Forecast Regression $\hat{b}$	Forecast Regression R^2
Two different volatilities	-3.86%	7.33%	0.86	89%
One volatility	0.29%	5.90%	0.92	93%
Volatility of 30%	0.27%	6.48%	0.88	91%

MK

	$A - F$ Mean	$A - F$ Standard Deviation	Forecast Regression $\hat{b}$	Forecast Regression R^2
Two different volatilities	-3.55%	6.76%	0.93	87%
One volatility	-0.01%	5.83%	1.01	91%
Volatility of 30%	-0.18%	5.87%	0.91	91%

Table 7b
Calibration and Forecasting Results for Individual Issues Using only One Volatility

	At Issuance								Pricing One Year Later			
Issuer	Issue Date	Tenor	Coupon	*S/X*	Volatility	TF Credit Spread	GS Credit Spread	MK Default Intensity	Convertible Market Price	TF (A - F)	GS (A - F)	MK (A - F)
Anthem, Inc.	10/2/2012	30.0	2.75%	82%	30%	1.5%	1.4%	2.8%	$130.19	-5.2%	-5.9%	-6.0%
Brocade Comm. Systems	1/9/2015	5.0	1.38%	74%	30%	2.4%	2.2%	4.0%	$93.50	1.9%	2.2%	2.1%
Cardtronics plc	11/21/2014	6.0	1.00%	75%	34%	4.3%	3.8%	8.4%	$97.63	-0.2%	0.3%	-1.2%
Cobalt International Energy, Inc.	5/8/2014	10.0	3.13%	75%	40%	7.4%	5.9%	14.3%	$78.63	1.9%	5.0%	4.2%
Cobalt International Energy, Inc.	12/11/2012	7.0	2.63%	72%	74%	13.2%	9.3%	26.0%	$86.81	-4.7%	0.8%	-3.8%
Euronet Worldwide, Inc.	10/30/2014	29.9	1.50%	75%	34%	2.7%	1.7%	5.6%	$129.19	7.7%	5.8%	5.1%
Forest City Realty Trust, Inc	7/13/2012	6.1	4.25%	65%	44%	10.7%	9.4%	21.6%	$110.00	3.5%	2.6%	2.1%
Hornbeck Offshore Services, Inc.	8/8/2012	7.1	1.50%	78%	50%	7.7%	6.1%	14.5%	$127.06	5.4%	3.9%	4.0%
iStar Inc.	11/13/2013	3.0	1.50%	72%	24%	2.5%	2.5%	4.5%	$103.19	0.8%	0.8%	0.8%
iStar Inc.	11/7/2012	4.0	3.00%	62%	40%	8.0%	7.4%	15.2%	$127.88	-3.1%	-3.9%	-4.3%
Lam Research Corporation	5/10/2012	4.0	0.50%	61%	41%	4.2%	3.9%	7.6%	$110.63	-5.4%	-5.5%	-5.7%
Lam Research Corporation	5/10/2012	6.0	1.25%	61%	41%	4.3%	3.9%	7.3%	$117.19	-9.1%	-9.5%	-8.4%
LinkedIn Corporation	11/6/2014	5.0	0.50%	78%	41%	4.4%	3.8%	8.7%	$108.50	1.9%	2.1%	0.8%
MGIC Investment Corp.	3/7/2013	7.1	2.00%	71%	131%	18.1%	10.4%	42.9%	$142.38	22.6%	13.2%	15.3%
Micron Technology, Inc.	11/6/2013	30.0	3.00%	64%	40%	6.9%	4.0%	16.4%	$126.50	14.3%	5.9%	7.6%
Radian Group Inc.	2/26/2013	6.0	2.25%	99%	70%	14.0%	9.1%	30.1%	$159.75	5.1%	2.1%	3.8%
Red Hat, Inc.	10/2/2014	5.0	0.25%	79%	31%	2.4%	2.2%	4.5%	$124.44	-6.3%	-6.4%	-6.9%
RPM International Inc.	12/3/2013	7.0	2.25%	76%	24%	0.0%	0.0%	0.0%	$113.38	3.7%	3.9%	3.7%
SanDisk Corp.	10/24/2013	7.0	0.50%	75%	29%	0.8%	0.8%	1.4%	$117.75	-3.4%	-3.5%	-3.3%
SEACOR Holdings Inc.	11/18/2014	14.0	3.00%	60%	20%	3.6%	3.5%	7.6%	$79.25	2.9%	3.6%	2.8%
Stone Energy Corp.	2/29/2012	5.0	1.75%	75%	64%	14.4%	11.0%	29.4%	$91.06	-11.3%	-7.1%	-10.1%
Stone Energy Corp.	5/8/2013	3.8	1.75%	52%	41%	5.0%	4.8%	9.0%	$121.44	-0.4%	-1.2%	-0.2%
Tesla Motors, Inc.	2/27/2014	7.0	1.25%	70%	58%	8.2%	6.3%	15.4%	$83.69	6.1%	8.1%	6.9%
Tesla Motors, Inc.	2/27/2014	5.0	0.25%	70%	58%	6.8%	5.5%	13.2%	$86.63	8.1%	9.5%	6.6%

Issuer	Issue Date	Tenor	Coupon	*S/X*	Volatility	TF Credit Spread	GS Credit Spread	MK Default Intensity	Convertible Market Price	TF (A - F)	GS (A - F)	MK (A - F)
Tesla Motors, Inc.	5/16/2013	5.0	1.50%	74%	53%	10.8%	8.8%	20.4%	$179.94	-12.6%	-16.7%	-13.1%
The Priceline Group Inc.	6/3/2014	6.0	0.35%	93%	24%	0.0%	0.0%	0.0%	$113.19	3.1%	3.1%	3.2%
The Priceline Group Inc.	3/6/2012	6.0	1.00%	69%	37%	3.5%	3.2%	6.3%	$110.63	-3.7%	-3.7%	-3.5%
The Priceline Group Inc.	5/29/2013	7.0	0.35%	60%	33%	2.0%	1.8%	3.6%	$120.81	-0.7%	-1.3%	-0.7%
The Priceline Group Inc.	3/8/2013	5.0	1.00%	76%	34%	2.1%	1.9%	3.6%	$148.31	9.1%	8.8%	8.6%
The Priceline Group Inc.	8/14/2014	7.1	0.90%	59%	24%	0.4%	0.4%	0.6%	$98.31	1.1%	1.1%	1.1%
TTM Technologies Inc.	12/16/2013	7.0	1.75%	86%	40%	7.6%	6.0%	16.4%	$96.44	-1.4%	0.1%	-0.9%
Twitter, Inc.	9/11/2014	5.0	0.25%	66%	54%	5.6%	4.7%	10.4%	$88.31	-4.8%	-3.0%	-5.3%
United States Steel Corp.	3/20/2013	6.0	2.75%	78%	45%	9.4%	7.9%	18.1%	$127.94	-4.5%	-5.5%	-5.7%
Verint Systems Inc.	6/12/2014	7.0	1.50%	78%	25%	2.5%	2.3%	4.6%	$116.06	1.1%	0.8%	1.5%
Yahoo! Inc.	12/5/2014	4.0	0.00%	94%	33%	2.9%	2.5%	5.3%	$98.25	-1.0%	-0.5%	-1.4%
Average		8.15	1.56%	73%	43%	5.71%	4.52%	11.42%	$113.28	0.65%	0.29%	-0.01%
Standard deviation										6.93%	5.90%	5.83%

Conclusions

In three important respects the convertible debt models are very similar. First, our base case example illustrated the three models produce quite similar prices for the same inputs and a jump in the stock price of -70% in the event of default. Second, we showed that there is little effect on the value of the convertible if we make the credit spread or default intensity a function of stock price. Third, we provide evidence that the three models perform well in forecasting convertible debt prices one-year after calibrating the each of models to market data. In the context of estimating convertible debt prices for non-traded securities for financial reporting purposes this is reassuring. We also identified one important difference in the comparative responses of the TF and GS models and the MK model to changes in credit quality. Contrary to expectations the convertible debt price produced by the MK model increased when credit quality decreased. This is because there is a complex relationship between credit quality and volatility of equity such that the decrease in credit quality increased stock price volatility and increased the value of the conversion feature. This observation highlights an important underlying difference in the assumptions of the TF and GS models as compared to the MK model. The TF and GS models assume that the stock price follows a geometric Brownian motion, while the MK model assumes the stock price follows a jump-diffusion process. This difference should be considered in estimating the volatilities to use in each model. In particular, volatility estimation is more complex if you assume stock prices follow a jump-diffusion process. That said, on the whole, we find that the models are sufficiently similar that all three can reasonably be used to estimate the value of non-traded convertible debt and will provide very similar valuations of the convertible debt. If you use these models to partition the debt component, the debt component will generally be largest for the TF model and smallest for the MK model.

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